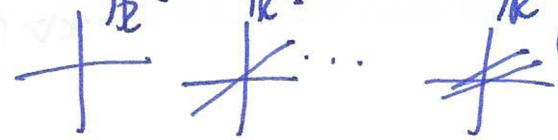
<u>Linear algebra</u> intuition $\mathbb{R}^n$ like $\mathbb{R}^3$: $\mathbb{R}$ ⟶ $\mathbb{R}^2$ $\mathbb{R}^3$ ... $\mathbb{R}^n$



consider $\mathbb{R}^n$ vs n-cube I^n vs n-ball B^n vs. $S^n = \partial B^{n+1}$
vs simplex Δ^n

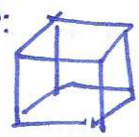
simplest case: discrete case: $\overset{\text{simplex}}{\Delta^0} = \cdot$ $\Delta^1 =$ •—• $\Delta^2 =$ Δ^3 ....

Q: how many vertices
Q: diameter?
Q: number of nearest neighbours.

<u>Cube</u>: (vertices/edges) C^0 . C^1 C^2 C^3

Q: how many vertices
Q: diameter? (combinatorial / in Euclidean metric)
Q: number of nearest neighbours.

<u>Geometry</u>: I^n:



what is $\mathrm{vol}(N_r(C^n) \subseteq C^{n+1})$?
Q: what is $\mathrm{vol}(C^n)$?
Q: what is (Euclidean) diameter?

B^n/S^n: Q: what is $\mathrm{vol}(B^n)$, $\mathrm{vol}(S^n)$?
Q: what is $\mathrm{vol}(N_r(B^n)) \leq N_r(B^{n+1})$?
Q: what is $\mathrm{vol}(N_r(S^n) \leq N_r(S^{n+1})$?
Q: what is $\mathrm{vol}(B^n) \leq (2I)^n$?

Answers: $|S_1^{n-1}| = \dfrac{2\pi^{n/2}}{\Gamma(\frac{n}{2})}$ $\quad |B_1^n| = \dfrac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)}$.

$$B_1^{n+1} = \int_{-1}^{+1} B_1^n r^n \, dx \qquad r = \sqrt{1-x^2}\,dx$$

$$= \int_{-1}^{1} B_1^n (1-x^2)^{n/2} dx \qquad x = \sin\theta$$

$$\frac{dx}{d\theta} = \cos\theta$$

$$= B_1^n \int_{-\pi/2}^{\pi/2} \cancel{B_1^n} (\cos^2\theta)^{n/2} \cdot \cos\theta \, d\theta$$

$$= 2 B_1^n \int_0^{\pi/2} \cos^{n+1}\theta \, d\theta .$$

evaluating $\int_0^{\pi/2} \cos^{n+1}\theta \, d\theta$. induction.

n=1 $\int_0^{\pi/2} \cos\theta \, d\theta = \left[\sin\theta\right]_0^{\pi/2} = 1$.

n=2 $\int_0^{\pi/2} \cos^2\theta \, d\theta$ double angle / parts.

$$\int_0^{\pi/2} \underset{u}{\cos\theta} \cdot \underset{v'}{\cos\theta} \, d\theta = \underbrace{\left[\cos\theta \cdot \sin\theta\right]_0^{\pi/2}}_{0} - \int_0^{\pi/2} -\sin\theta \cdot \sin\theta \, d\theta$$

$$= \int_0^{\pi/2} \sin^2\theta \, d\theta = \int_0^{\pi/2} 1 - \cos^2\theta \, d\theta.$$

$$2\int_0^{\pi/2} \cos^2\theta \, d\theta = \int_0^{\pi/2} 1 \, d\theta = \left[\theta\right]_0^{\pi/2} = \frac{\pi}{2}.$$

$$\int_0^{\pi/2} \cos^2\theta \, d\theta = \frac{\pi}{4}.$$

in general:

$$\int_0^{\pi/2} \cos^n\theta \cos\theta \, d\theta = \underbrace{\left[\cos^n\theta \sin\theta\right]_0^{\pi/2}}_{0} - \int_0^{\pi/2} n\cos^{n-1}\theta \cdot -\sin\theta \sin\theta \, d\theta$$

$$= n\int_0^{\pi/2} \cos^{n-1}\theta \cdot \sin^2\theta \, d\theta.$$

$$= n\int_0^{\pi/2} \cos^{n-1}\theta \left(1 - \cos^2\theta\right) d\theta.$$

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$$(n-1)\int_0^{\pi/2} \cos^{n-1}\theta \, d\theta = \int_0^{\pi/2} \cos^{n-1}\theta \, d\theta \quad \ldots$$