$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \iff 1 + \cot^2 x = \operatorname{cosec}^2 x$$

<u>Double angle</u>

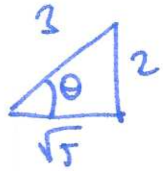
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

<u>Addition</u>

$$\sin(A+B) = \sin A\cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A \sin B$$

<u>special case</u> (shift) $\sin\left(x + \frac{\pi}{4}\right) = \cos(x)$

<u>Example</u> suppose $\sin\theta = \frac{2}{3}$, find $\cos\theta$, $\tan\theta$, $\sin 2\theta$



$\cos\theta = \frac{\sqrt{5}}{3}$ $\tan\theta = \frac{2}{\sqrt{5}}$ $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2}{3}\cdot\frac{\sqrt{5}}{3} = \frac{4\sqrt{5}}{9}$

§1.5 Inverse functions

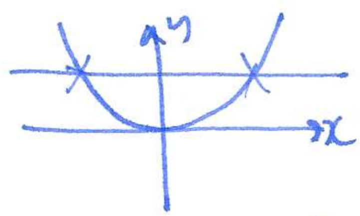
<u>recall</u>: $f: \mathbb{R} \to \mathbb{R}$ ← range
$x \mapsto f(x)$
domain

want: the inverse function should be the reverse of this

$$\mathbb{R} \xleftarrow{f^{-1}} \mathbb{R}$$
$$f^{-1}(x) \leftarrow x$$

<u>problem</u>: inverse might not be a function.

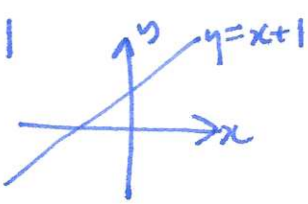
<u>example</u> $f: \mathbb{R} \to \mathbb{R}$
$x \mapsto x^2$



$f(+1) = 1$
$f(-1) = 1$
Q: what is $f^{-1}(1)$?

Q: when does a function have an inverse?
A: when it passes the <u>horizontal line test</u> (one-to-one | injective)
$\iff$ for each element $c \in$ range, there is at most one $x \in$ domain such that $f(x) = c$

<u>Example</u> $y = x+1$



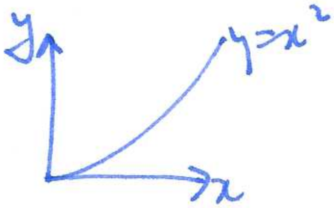
Q: how do we find a formula for the inverse?
A: · write down $y = f(x)$
· solve for x in terms of y, i.e. $x = g(y)$

- $f^{-1}(x) = g(x)$
- check! $f(f^{-1}(x)) = x = f^{-1}(f(x))$

$y = x+1$
$y-1 = x$
inverse $f^{-1}(x) = x-1$

<u>Example</u> $f(x) = x^2$ <u>problem</u>: no inverse! (doesn't pass horizontal line test)

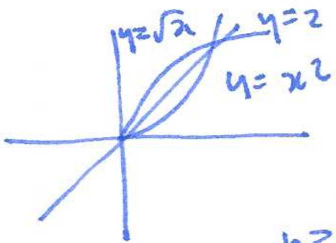
<u>fix</u>: restrict domain, consider $f: [0,\infty) \to [0,\infty)$
$x \mapsto x^2$



now <u>does</u> pass the horizontal line test
so has an inverse we call $\sqrt{x}$ $f^{-1}: [0,\infty) \to [0,\infty)$
$x \mapsto \sqrt{x}$

<u>How to draw the graph of the inverse</u>

- reflect in $y=x$ reason: graph is pairs $(x, f(x))$
(input, output)



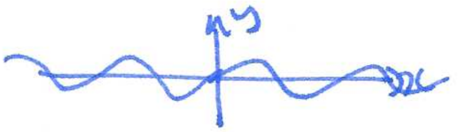
graph of f^{-1} is pairs $(f(x), x)$

$y = f(x) \leftrightarrow f^{-1}(y) = x$ $(x, f(x)) \leftrightarrow (f^{-1}(y), y)$
swap and relabel

<u>Inverse trig functions</u> <u>problem</u>: not one-to-one

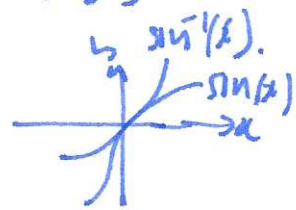
$y = \sin(x)$

<u>fix</u>: restrict domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

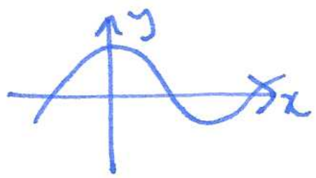


$\sin(x): [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1,1]$

$\arcsin(x) = \operatorname{asin}(x) = \sin^{-1}(x): [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



similarly $y = \cos(x)$

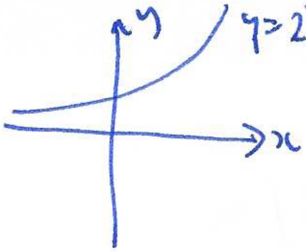


restrict domain to $[0,\pi]$
$\cos(x): [0,\pi] \to [-1,1]$
$\cos^{-1}(x): [-1,1] \to [0,\pi]$

<u>§1.6 Exponential and logarithm functions</u>

Example: $x \mapsto 2^x$

x	-2	-1	0	1	2	3	4
2^x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



can use any positive number apart from 2, $f(x) = b^x$ ⑦
$b > 0$

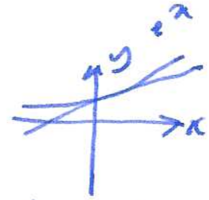
useful properties:
- positive $(b>0)$
- b^x increasing if $b>1$
  decreasing if $b<1$
- b^x grows faster than any polynomial

<u>exponent rules</u> $b^0 = 1$ $b^x b^y = b^{x+y}$ $b^{-x} = \frac{1}{b^x}$ $\frac{b^x}{b^y} = b^{x-y}$

$(b^x)^y = b^{xy}$ $b^{1/n} = \sqrt[n]{b}$

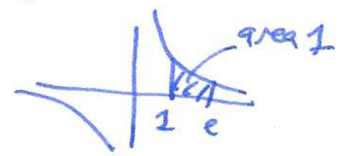
- there is a special exponential function e^x, $e = 2.71828\ldots$

<u>key properties of e^x</u>



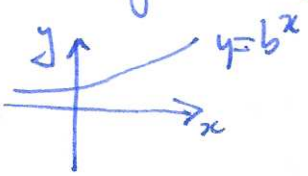
① e is the unique number s.t. e^x has slope 1 at $x=0$

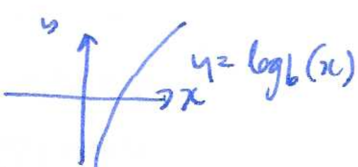
② e is the unique number s.t. the area under the curve $y=\frac{1}{x}$ between 1 and e is equal to 1.



<u>Logarithms</u>.

the logarithm is the inverse function of the exponential function.





the special logarithm function with $b=e$ is called the <u>natural logarithm</u> $\ln(x)$

- inverse function properties $f^{-1}(f(x)) = x = f(f^{-1}(x))$

$b^{\log_b x} = x = \log_b(b^x)$

- logarithm rules: $\log_b(1) = 0$ $\log_b(b) = 1$

$\log_b(st) = \log_b(s) + \log_b(t)$ $\log_b\left(\frac{1}{t}\right) = -\log_b(t)$

$\log_b\left(\frac{s}{t}\right) = \log_b(s) - \log_b(t)$ $\log_b(s^t) = t\log_b(s)$

base conversion: $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$ for any a, so $\log_b(x) = \frac{\ln(x)}{\ln(b)}$