MTH 231 Calculus I

Joseph Maher joseph.maher@csi.cuny.edu

web page: http://www.math.csi.edu/~maher

office 1S-222 office hours M 12:20-2:15 (in 1S-222) W 12:20-1:10 (in 1S-214)

- math tutoring 1S-214
- students w/disability ← make sure I know what to do

Text: Calculus, early transcendentals, Rogawski + Adams.

HW: webworks

§1.2 Linear and quadratic functions

recall: input set (domain) $\xrightarrow{\text{function}}$ output set (range)

examples $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ ($\mathbb{R}$ = set of real numbers) or $f(x) = x^2$

example values: $0 \mapsto 0$ $-1 \mapsto 1$ $1 \mapsto 1$ $2 \mapsto 4$ etc.

notation: $f: \mathbb{R} \to \mathbb{R}$

name, inputs/domain, outputs/range

Example
- $+: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $(a,b) \mapsto a+b$
- evaluation at 0: $\{\text{functions } f: \mathbb{R} \to \mathbb{R}\} \to \mathbb{R}$, $f \longmapsto f(0)$

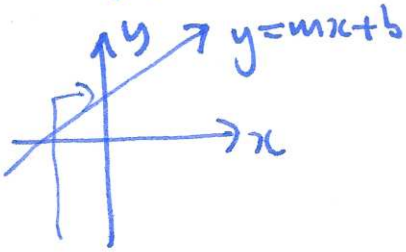
key property: each input x gets mapped to a single output $f(x)$, not multiple outputs

A linear function $f: \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx+b$

(m, b "constants", i.e. don't depend on x)

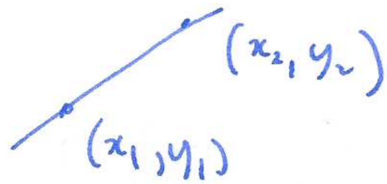
examples
$f(x) = 0$ $f(x) = -3x+2$
$f(x) = x$ $f(x) = ax+b$

The graph of a linear function is a straight line



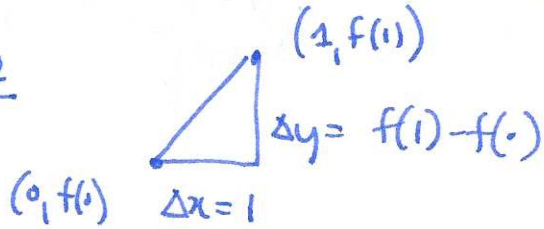
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

$f(0) = m \cdot 0 + b$ "y-intercept"



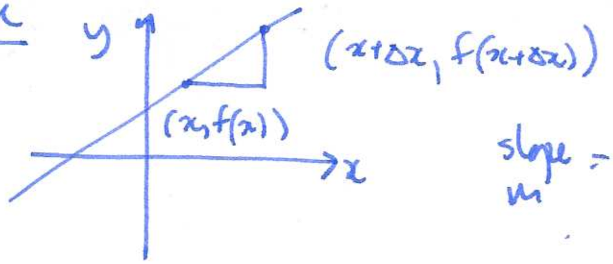
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

special case



$$\underset{m}{\text{slope}} = \frac{\Delta y}{\Delta x} = \frac{f(1) - f(0)}{1 - 0} = \frac{m + b - b}{1} = m$$

general case



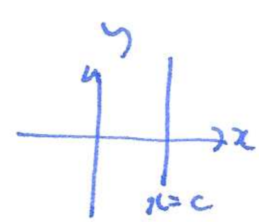
$$\underset{m}{\text{slope}} = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$$

$$= \frac{m(x + \Delta x) + b - mx + b}{\Delta x} = \frac{m \Delta x}{\Delta x} = m.$$

useful fact: a straight line has constant slope everywhere

observations:

- $|m|$ large $\Leftrightarrow$ steep slope
- $m = 0$ horizontal line
- $m > 0$ increasing
- $m < 0$ decreasing
- vertical lines not graphs of functions



equation vs function

↑ relation between variables

$x = c$

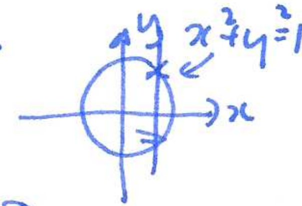
$x^2 + y^2 = 1$

↑ a map from one set to another

e.g. $f : \mathbb{R} \to \mathbb{R}$

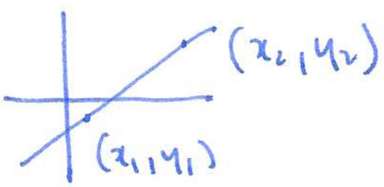
$f(x) = x^2 + 2$

the graph of $f:\mathbb{R}\to\mathbb{R}$ is ~~$y=f(x)$~~ pairs $(x,f(x)) \subseteq \mathbb{R}^2$, satisfies $y=f(x)$ (an equation!) but not all equations come from functions.



to deal with any straight line, use the general/symmetric linear equation $ax+by=c$, at least one of a,b not zero.

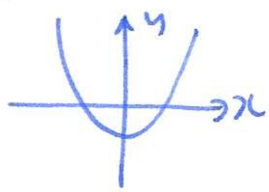
useful technique: find equation of line through two points.



- find slope $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2-y_1}{x_2-x_1}$
- point slope formula for a line $y-y_1 = m(x-x_1)$

quadratic functions given by $f(x) = ax^2+bx+c$ (a,b,c constants, do not depend on x)

graphs are parabolas



at most two distinct real solutions to $f(x)=0$ given by $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac<0$ $\qquad$ $b^2-4ac=0$ $\qquad$ $b^2-4ac>0$

useful techniques

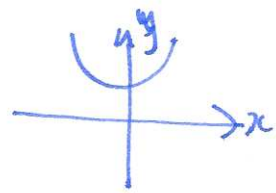
- factorization: if $ax^2+bx+c = a(x-r_1)(x-r_2)$ then r_1, r_2 are solutions/roots
- complete the square: any quadratic can be written as $a(x+b)^2+c$

example:

x^2+2x+3

$(x+1)^2 \quad +2$

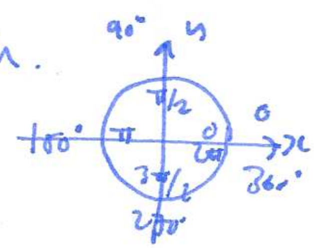
$x^2+2x+1+2$



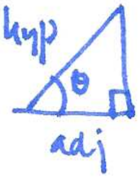
← no solutions!

§1.4 Trig functions

- angles vs radians, radians win.
- angle in radians = distance travelled around the unit circle



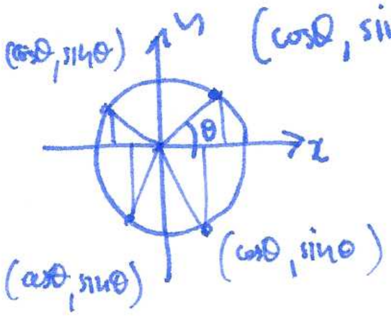
- right angled triangles



$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$

can extend these functions to be defined for all $\theta \in \mathbb{R}$



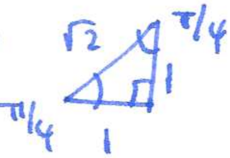
useful facts: $\sin(-\theta) = -\sin\theta$ (odd function)

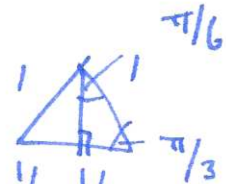
$\cos(-\theta) = \cos\theta$ (even function)

special values :

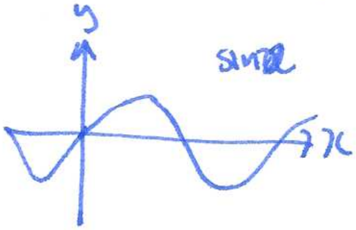
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

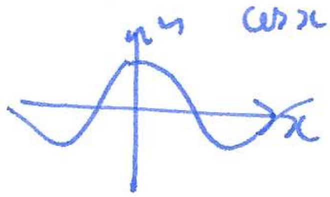
this comes from special triangles





graphs of $\sin(\theta), \cos(\theta)$ periodic functions of period 2π





More Trig functions $\tan\theta = \dfrac{opp}{hyp} = \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{1}{\sin\theta} = \operatorname{cosec}\theta$

$\dfrac{1}{\cos\theta} = \sec\theta$

$\dfrac{1}{\tan\theta} = \cot(\theta) = \dfrac{\cos\theta}{\sin\theta}$

Trig identities Pythagorean identity $\sin^2 x + \cos^2 x = 1$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \leftrightarrow \tan^2 x + \cancel{\cos^2 x}\, 1 = \sec^2 x$$