

MTH 306 History of Math, Fall 2025

For Wed Oct 22nd

- (1) Read handout 8.

HW6 Due Wed Oct 29th

- (1) Solve $ax^2 + bx + c = 0$ by completing the square, and observe that this gives the standard formula for solving quadratic equations.
- (2)
 - (a) Show that $ax^3 + bx^2 + cx + d = 0$ can be reduced to the form $x^3 + Px = Q$ by dividing by a and completing the cube.
 - (b) Show $(a - b)^3 + 3ab(a - b) = a^3 - b^3$.
 - (c) Set $x = a - b$, $P = 3ab$ and $Q = a^3 - b^3$, and then write $(a^3 + b^3)^2$ in terms of P and Q .
 - (d) Solve the pair of equations for $a^3 + b^3$ and $a^3 - b^3$ to give a^3 and b^3 in terms of P and Q .
 - (e) Finally find an expression for $x = a - b$. (This is Tartaglia/Cardan's method.)
- (3) A general linear equation in two variables is of the form $ax + by + c = 0$. The equation is non-degenerate if at least one of a or b is non-zero. Show that you can use a change of coordinates to turn a non-degenerate linear equation into $y = 0$.
- (4) A general quadratic equation in two variables is of the form $ax^2 + bxy + cy^2 + dx + ey + f = 0$. The equation is non-degenerate if at least one of a, b or c is non-zero. Find explicit examples of non-degenerate quadratic equations with the following solution sets.
 - (a) Empty
 - (b) A single point
 - (c) A single straight line
 - (d) A pair of parallel lines
 - (e) A pair of intersecting lines
 - (f) A circle
 - (g) A parabola
 - (h) A hyperbola
- (5) Show that you can use a change of coordinates to take any non-degenerate quadratic equation to one of the above. (This is the classification of conics.)