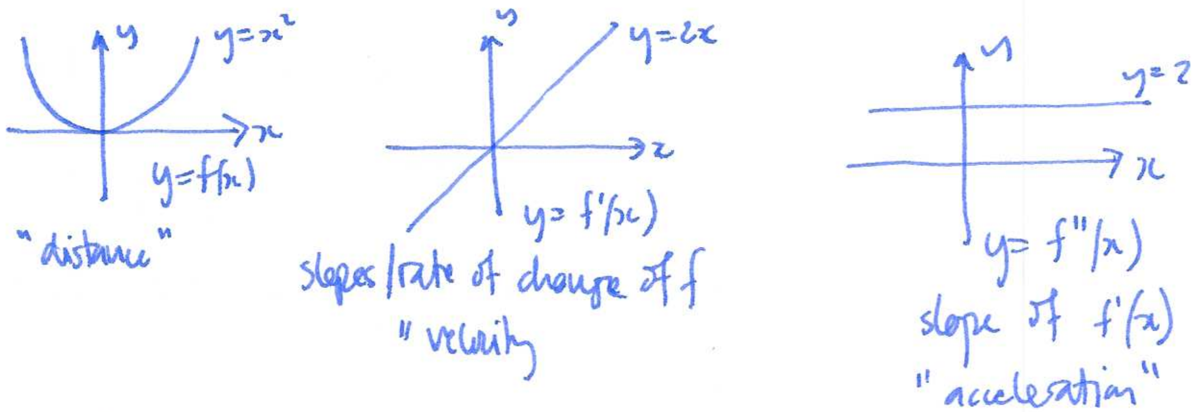
$s(t) = 2 + 10t - \frac{1}{2}gt^2$

$v(t) = 10 - gt$ $\quad v(t) = 0 \Rightarrow t = \frac{10}{g} \sim 1 \quad s(1) = 2 + 10 - 5 = 7m.$

- how fast is it going when it hits the ground?
- if I can throw a stone 10m high, how fast can I throw it?

§3.5 Higher derivatives



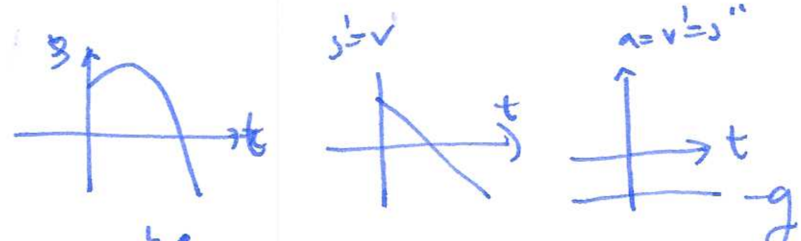
Example $f(x) = xe^x$

$f'(x) = xe^x + e^x$

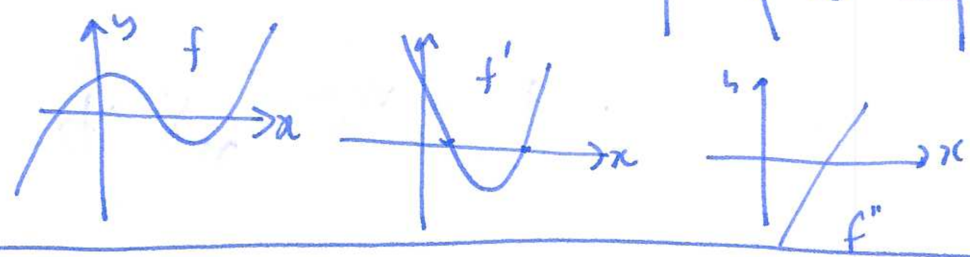
$f''(x) = xe^x + e^x + xe^x = xe^x + 2e^x$

$f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$ etc.

Example acceleration due to gravity



Example



Examples ① $e^{4x} = f(g(x))$ where $f(x) = e^x$ $\quad f'(x) = e^x$

$g(x) = 4x \quad g'(x) = 4$

so $(e^{4x})' = f'(g(x)) \cdot g'(x) = e^{4x} \cdot 4$

② $\sin^2(x) = f(g(x))$ where $f(x) = x^2 \quad f'(x) = 2x \quad g(x) = \sin x \quad g'(x) = \cos x$

§3.6 Trigonometric functions

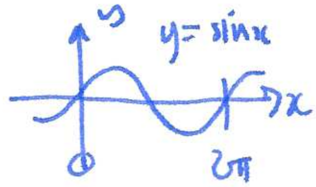
Thm $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

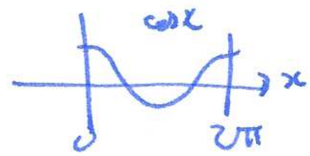
recall
$\sin(A+B) =$
$\sin A \cos B + \cos A \sin B$

Proof (for $\sin x$) $\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} = \lim_{h \to 0} \sin x \cdot \underbrace{\frac{\cos h - 1}{h}}_{\to 0} + \cos x \underbrace{\frac{\sin h}{h}}_{\to 1} = \cos x \quad \square$$

Q: can this be right?





Example $f(x) = x \sin x$

$f'(x) = x\cos x + \sin x$

Thm $\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x \qquad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Proof (of $\tan x$) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x - \sin x \sin x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$= \sec^2 x \quad \square$

Example $\frac{d}{dx}(e^x \cos x) = e^x \cdot (-\sin x) + e^x \cos x$

§3.7 Chain rule

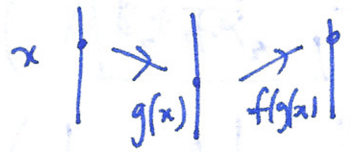
composition of functions $f(g(x)) = (f \circ g)(x)$

Examples e^{4x}, $\sin^2 x$, etc....

Thm If f, g differentiable functions, then $f \circ g$ is differentiable and

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$ mnemonic: $(f(g(x)))' =$ outside$'$(inside) . (inside)$'$

Note: $f(g(x))$ $\quad \mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$



stretch factor $\quad g'(x) \quad f'(g(x))$

Examples ① $e^{4x} = f(g(x))$ where $f(x) = e^x \quad f'(x) = e^x$
$g(x) = 4x \quad g'(x) = 4$

so $(e^{4x})' = f'(g(x)) \cdot g'(x) = e^{4x} \cdot 4$

② $\sin^2(x) = f(g(x))$ where $f(x) = x^2 \quad f'(x) = 2x$
$g(x) = \sin(x) \quad g'(x) = \cos(x)$

so $(\sin^2(x))' = 2\sin x \cos x$.

③ $\sqrt{x^3+1}$ etc.

Alternate notation $f(g(x)) \leftrightarrow f(u), \quad u = g(x)$

$$\frac{df}{dx} = f'(u)\frac{du}{dx} = \frac{df}{du}\frac{du}{dx} \qquad \boxed{\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}}$$

mnemonic: "cancelling fractions".

Examples $\cos(x^2)$, $e^{\sqrt{x}}$, $\sin\left(\frac{\pi x}{180}\right)$, $\sqrt{x+\sqrt{x^2+1}}$.

Proof (of chain rule)

$$[f(g(x))]' = \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{\cancel{g(x+h)}\ h} \qquad [\text{answer should be } f'(g(x))g'(x)]$$

write this as $$\lim_{h\to\infty} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h}$$

set $k = g(x+h) - g(x)$, as g is continuous, $h \to 0 \Rightarrow k \to 0$

so $$= \lim_{k\to 0} \frac{f(g(x)+k) - f(g(x))}{k} \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} = f'(g(x)) \cdot f'(x). \quad \square$$