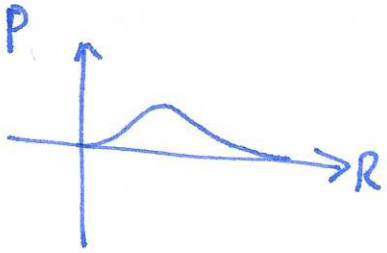
③ application: battery power

R resistance

r internal resistance

power $P = \frac{V^2 R}{(r+R)^2}$



Q: when does the battery give maximal power?

A: when $\frac{dP}{dR} = 0$ $\quad P = \frac{V^2 R}{(r+R)^2}$ $\quad P(R)$, V, r constant

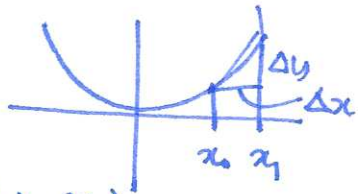
$$\frac{dP}{dR} = \frac{(r+R)^2 (V^2 R)' - ((r+R)^2)' V^2 R}{(r+R)^4} = \frac{(r+R)^2 V^2 - (r^2+2rR+R^2) V^2 R}{(r+R)^4}$$

$$= V^2 \frac{r^2+2rR+R^2-(2r+2R)R}{(r+R)^4} = V^2 \frac{r^2-R^2}{(r+R)^4} = V^2 \frac{r-R}{(r+R)^4}$$

$\frac{dP}{dR} = 0$ when $r = R$. □

§2.4 Rates of change

recall: average rate of change $\frac{\Delta y}{\Delta x} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$



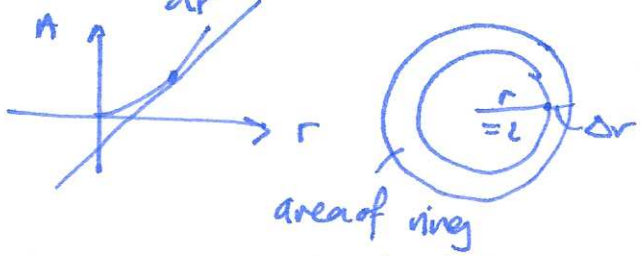
instantaneous rate of change: $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1-x_0}$

observation: if Δx is small, we can use the average rate of change to approximate the actual rate of change, and vice versa.

Example area of circle is πr^2 $\quad A = \pi r^2$

calculate the rate of change of area wrt radius $\frac{dA}{dr} = 2\pi r$

e.g. $\left.\frac{dA}{dr}\right|_{r=2} = 4\pi$ $\quad \left.\frac{dA}{dr}\right|_{r=5} = 10\pi$



for small $\Delta r, h$: $f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}$

so $f(x+h) \approx f(x_0) + h f'(x_0)$

$f(x_0+h) \approx f(x_0) + hf'(x_0)$ ← linear approximation formula

Example stopping distance in feet given by $F(s) = 1.1s + 0.05s^2$ (s = speed in mph) calculate stopping distance when $s=30$ $\quad F(30) = 78$

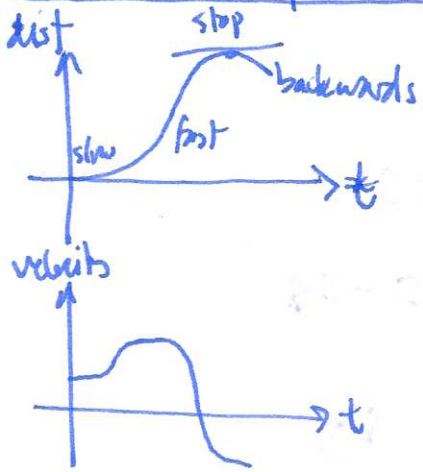
$F'(s) = 1.1 + 0.1s \quad F'(30) = 1.1 + 0.1 \times 30 = 4.1$ ft/mph

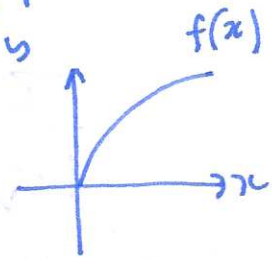
estimate stopping distance at $s=31$ (using above info)

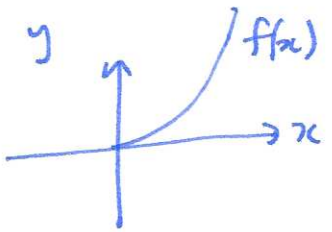
$F(s+h) \approx F(s) + hF'(s)$

$F(31) \approx F(30) + 1 \cdot F'(30) = 78 + 4.1 = 82.1$

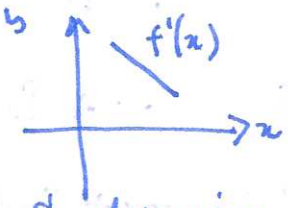
On the interpretation of graphs



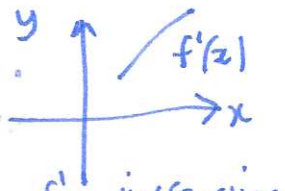




increasing functions

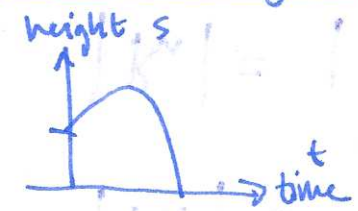


f' decreasing



f' increasing

Motion under gravity



$s_0 = s(0) =$ height at $t=0$

$V_0 = s'(0) =$ velocity at $t=0$

$s''(t) = v'(t) = a(t) = -g \qquad g = 9.8 \text{ m/s}^2$ (constant) $= 32 \text{ ft/s}^2$

$s'(t) = v(t) = -gt + V_0$

$s(t) = -\frac{1}{2}gt^2 + V_0 t + s_0$

Example throw a stone upwards at 10m/s from height 2m, what is max height?