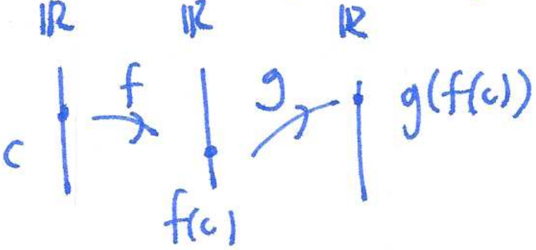
Thm 4 (inverse functions) If $f: D \to \mathbb{R}$ is continuous, with inverse $f^{-1}: \mathbb{R} \to D$ then f^{-1} is continuous.

Thm 5 (composition) If $f(x)$ is continuous at $x=c$ and $g(x)$ is continuous at $x=f(c)$, then $g(f(x))$ is continuous at $x=c$



Example $f(x) = \dfrac{2^x + \sin(x)}{\sqrt{x^2+x+1}}$ cts at $x=1$

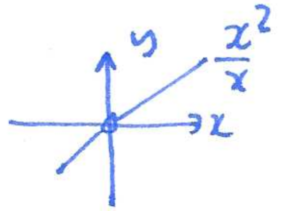
Q: where is $f(x) = \dfrac{x^2}{\sin(x)}$ continuous?

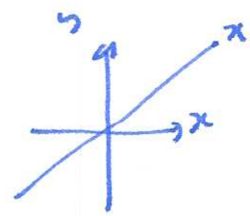
§2.5 Evaluating limits algebraically

Example $\dfrac{x^2}{x}$ undefined at $x=0$ $\quad \dfrac{0}{0} \leftarrow$ indeterminate form

but $\lim\limits_{x\to 0} \dfrac{x^2}{x}$ does not depend on value at $x=0$

$\dfrac{x^2}{x} = x$ for $x \neq 0$





so $\lim\limits_{x\to 0} \dfrac{x^2}{x} = \lim\limits_{x\to 0} x = 0$

indeterminate forms: $\dfrac{0}{0} \quad \dfrac{\infty}{\infty} \quad \infty \cdot 0 \quad \infty - \infty \quad 0^0$

note: $\dfrac{1}{0}$ not indeterminate form, limit will be $\pm\infty$ or DNE.

Examples ① $\lim\limits_{x\to 3} \dfrac{x^2-4x+3}{x^2+x-12}$ $\quad x=3: \quad \dfrac{9-12+3}{9+3-12} = \dfrac{0}{0}$

factor: $\dfrac{(x-3)(x-1)}{(x-3)(x+4)} = \dfrac{x-1}{x+4} \quad (x \neq 3)$

$\lim\limits_{x\to 3} \dfrac{(x-3)(x-1)}{(x-3)(x+4)} = \lim\limits_{x\to 3} \dfrac{x-1}{x+4} = \dfrac{2}{7}$

(2) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$ $\quad x=4: \quad \frac{2-2}{4-4} = \frac{0}{0}$

$$\frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)} = \frac{1}{\sqrt{x}+2} \qquad (x\neq 4)$$

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$$

(3) $\lim_{x\to \frac{\pi}{2}} \frac{\tan x}{\sec x} = \lim_{x\to\frac{\pi}{2}} \frac{\sin x/\cos x}{1/\cos x} = \lim_{x\to\frac{\pi}{2}} \sin x = 1$

(4) $\lim_{x\to 1} \frac{1}{x-1} - \frac{2}{x^2-1} = \frac{x+1-2}{x^2-1} = \frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1} \quad (x\neq 1)$

$$\lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$$

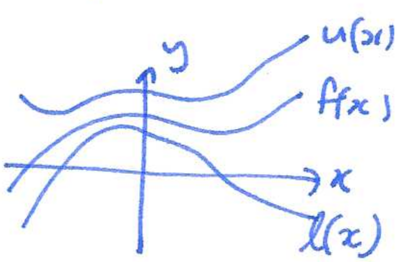
§2.6 Trigonometric limits

Q: what is $\lim_{x\to 0} \frac{\sin x}{x}$? $\quad x=0$ get $\frac{0}{0}$ indeterminate form

A: $\lim_{x\to 0} \frac{\sin x}{x} = 1$ (try ± 0.1, ± 0.01, ± 0.001, ...)

squeeze thm Thm: suppose $l(x) \leq f(x) \leq u(x)$

and $\lim_{x\to c} l(x) = L = \lim_{x\to c} u(x)$. then $\lim_{x\to c} f(x) = L$



Example $\lim_{x\to 0} x\sin\frac{1}{x}$