

MTH 231 Calculus I

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office 15-222 office hours M: 2:30-4:25 W 2:30-3:20 (in 15-214)

- math tutoring 15-214

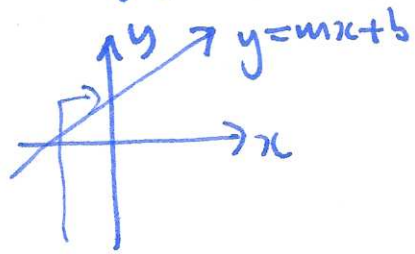
- students w/ disabilities

Text: Calculus, early transcendental, Rogawski + Adams

Hw: webworks

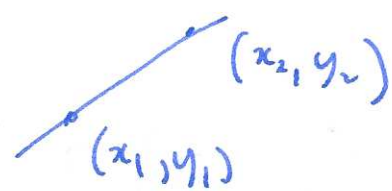
§1.2 Linear and quadratic functionsrecall: input set $\xrightarrow{\text{function}}$ output set
(domain) (range).examples $f: \mathbb{R} \rightarrow \mathbb{R}$ ($\mathbb{R}$ = set of real numbers)
 $x \mapsto x^2$
or $f(x) = x^2$ example values: $0 \mapsto 0$ $-1 \mapsto 1$
 $1 \mapsto 1$ $2 \mapsto 4$ etc.notation: $f: \mathbb{R} \rightarrow \mathbb{R}$
name $\nearrow$ domain/inputs $\nwarrow$ range/outputsExample $+: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$
 $(a, b) \mapsto a+b$ • evaluation at 0: $\{\text{functions}\} \rightarrow \mathbb{R}$
 $f: \mathbb{R} \rightarrow \mathbb{R}$ key property each input x gets mapped to a single output $f(x)$,
not multiple outputs $f \mapsto f(x)$ A linear function $f: \mathbb{R} \rightarrow \mathbb{R}$ has the form $f(x) = mx + b$
(m, b "constants", i.e. don't depend on x)

The graph of a linear function is a straight line



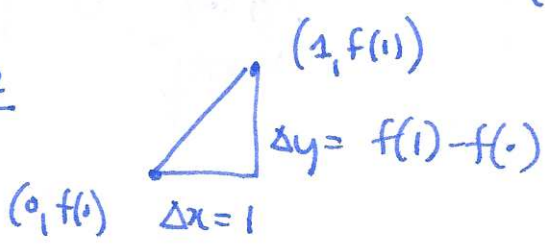
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

$f(0) = m \cdot 0 + b$ "y-intercept"



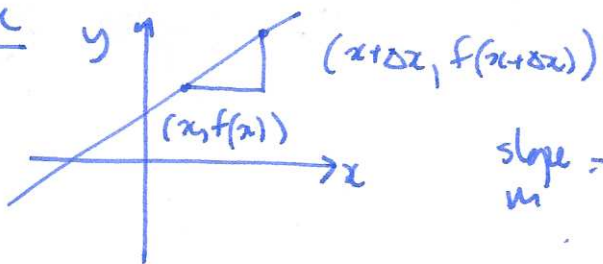
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

special case



$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{f(1) - f(0)}{1 - 0} = \frac{m + b - b}{1} = m$$

general case



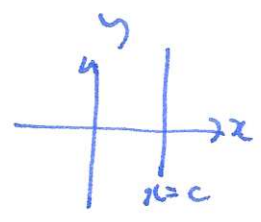
$$\text{slope } m = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x}$$

$$= \frac{m(x + \Delta x) + b - mx + b}{\Delta x} = \frac{m \Delta x}{\Delta x} = m$$

useful fact: a straight line has constant slope everywhere

observation:

- $|m|$ large $\leftrightarrow$ steep slope
- $m = 0$ horizontal line
- $m > 0$ increasing
- $m < 0$ decreasing
- vertical lines not graphs of functions



equation

vs function

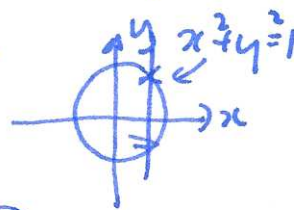
↑
relation between variables
 $x = c$
 $x^2 + y^2 = 1$

↑
a map from one set to another
e.g. $f: \mathbb{R} \rightarrow \mathbb{R}$
 $f(x) = x^2 + 2$

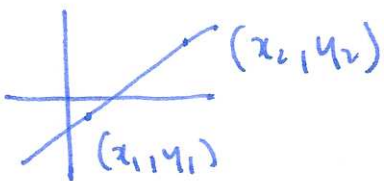
the graph of $f: \mathbb{R} \rightarrow \mathbb{R}$ is ~~$y=f(x)$~~ pair $(x, f(x)) \in \mathbb{R}^2$, satisfies $y=f(x)$ ③

(an equation!) but not all equations come from functions.

to deal with any straight line, use the general/symmetric
linear equation $ax+by=c$, at least one of a, b not zero.



useful technique: find equation of line through two points.



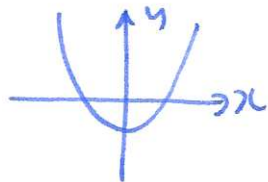
• find slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

• point slope formula for a line $y - y_1 = m(x - x_1)$

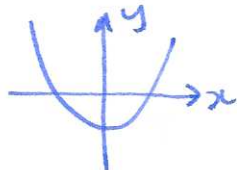
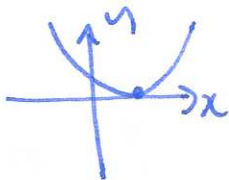
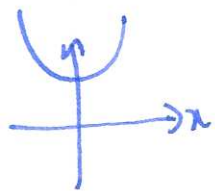
quadratic functions

graphs are parabolas

given by $f(x) = ax^2 + bx + c$ (a, b, c constants, do not depend on x)



at most two distinct real solutions
to $f(x)=0$ given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



$b^2 - 4ac < 0$

$b^2 - 4ac = 0$

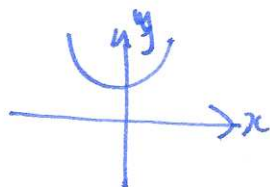
$b^2 - 4ac > 0$

useful techniques

- factorization: if $ax^2 + bx + c = a(x - r_1)(x - r_2)$ then r_1, r_2 are solutions/roots
- complete the square: any quadratic can be written as $a(x + b)^2 + c$

example:

$$\begin{aligned} x^2 + 2x + 3 \\ (x+1)^2 + 2 \\ x^2 + 2x + 1 + 2 \end{aligned}$$

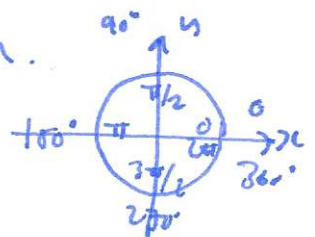


← no solutions!

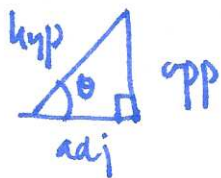
§1.4 Trig functions

• angles vs radians, radians win.

• angle in radians = distance travelled around the unit circle



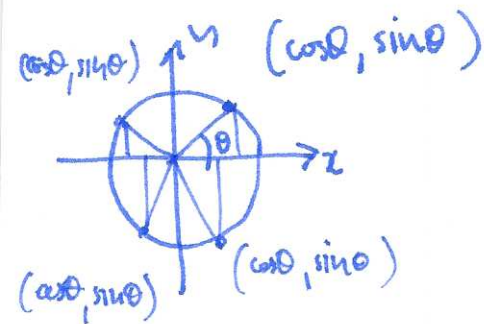
• right angled triangles



$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

can extend these functions to be defined for all $\theta \in \mathbb{R}$ ④



useful facts:

$$\sin(-\theta) = -\sin \theta$$

(odd function)

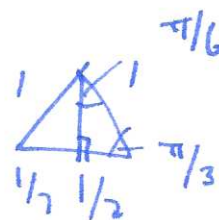
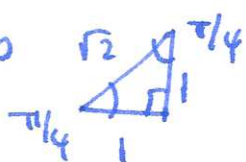
$$\cos(-\theta) = \cos \theta$$

(even function)

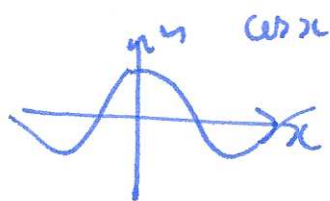
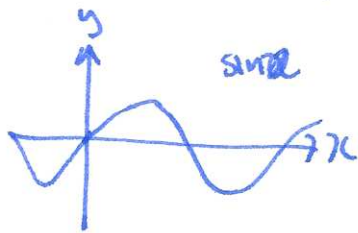
special values :

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos \theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

this comes from special triangles



graphs of $\sin(\theta), \cos(\theta)$ periodic functions of period 2π



More Trig functions

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$$

$$\frac{1}{\sin \theta} = \csc \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\frac{1}{\tan \theta} = \cot(\theta) = \frac{\cos \theta}{\sin \theta}$$

Trig identities

Pythagorean identity

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \Leftrightarrow \tan^2 x + 1 = \sec^2 x$$