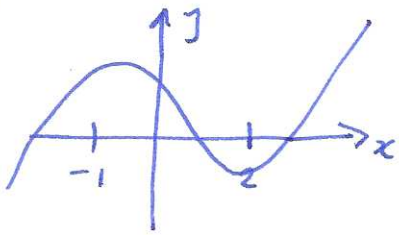
Thm e^x grows faster than any polynomial

Proof $\lim_{x\to\infty} \frac{e^x}{x^n} = \lim_{n\to\infty} \frac{e^x}{nx^{n-1}} = \cdots = \lim_{n\to\infty} \frac{e^x}{n!} = \infty$. $\square$.

§4.6 Graph sketching and asymptotes

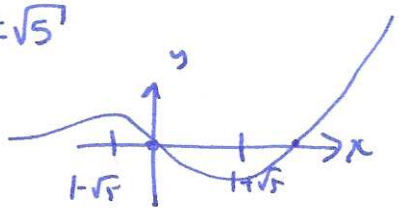
Example sketch graph of $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 3$.



useful info:
- critical points $f'(x) = x^2 - x - 2$, $(x-2)(x+1)$
- sign of $f'(x)$
- sign of $f''(x)$ $\quad f''(x) = 2x - 1$

critical points at $x = -1, 2$ $\quad f''(-1) = -3 < 0$ local max; $f''(2) = 3 > 0$ local min

Example $f(x) = (4x - x^2)e^x$, $x(4-x)$ $\quad$ critical points $x = 1 \pm \sqrt{5}$

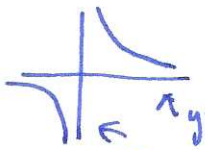


$f'(x) = (4 + 2x - x^2)e^x$

$f''(x) = (6 - x^2)e^x$

$1+\sqrt{5}$ local max; $1-\sqrt{5}$ local min

Asymptotes Example $y = \frac{1}{x}$



$y = 0$ horizontal asymptote

$x = 0$ vertical asymptote.

Defn $x = c$ is a **vertical asymptote** if $\lim_{x\to c^{\pm}} f(x) = \pm\infty$.

$y = c$ is a **horizontal asymptote** if $\lim_{x\to\pm\infty} f(x) = c$

Observation: rational functions $\frac{p(x)}{q(x)}$ have horizontal asymptotes if $\deg p \le \deg q$

Example $f(x) = \frac{x^2+x+1}{3x^2+2} \sim \frac{x^2}{3x^2} \to \frac{1}{3}$ as $x \to \infty$.