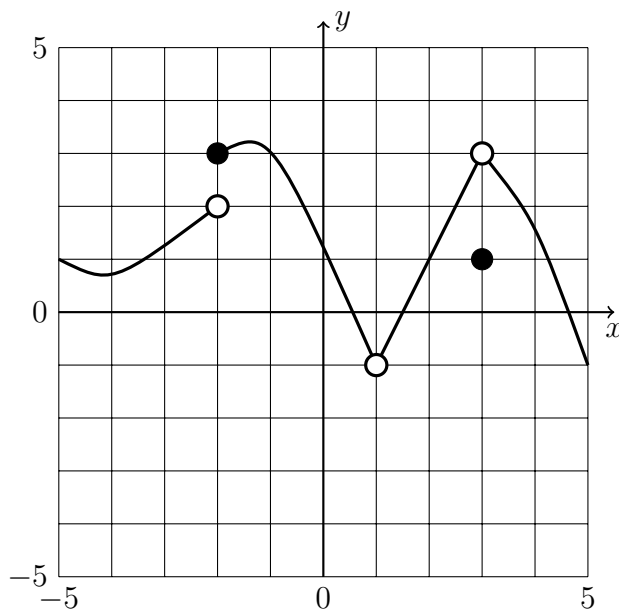


Math 231 Calculus 1 Fall 24 Sample Midterm 1

- (1) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



- (a) $\lim_{x \rightarrow -2^-} f(x)$
(b) $\lim_{x \rightarrow -2^+} f(x)$
(c) $\lim_{x \rightarrow -2} f(x)$
(d) $\lim_{x \rightarrow 1^-} f(x)$
(e) $\lim_{x \rightarrow 1^+} f(x)$
(f) $\lim_{x \rightarrow 3} f(x)$

- (2) Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

(a) $\lim_{x \rightarrow -2} \frac{x+2}{x^2-x-6}$

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{3+2x^2}}{2+3x}$

(c) $\lim_{x \rightarrow 0} \frac{\sin 4x}{5x}$

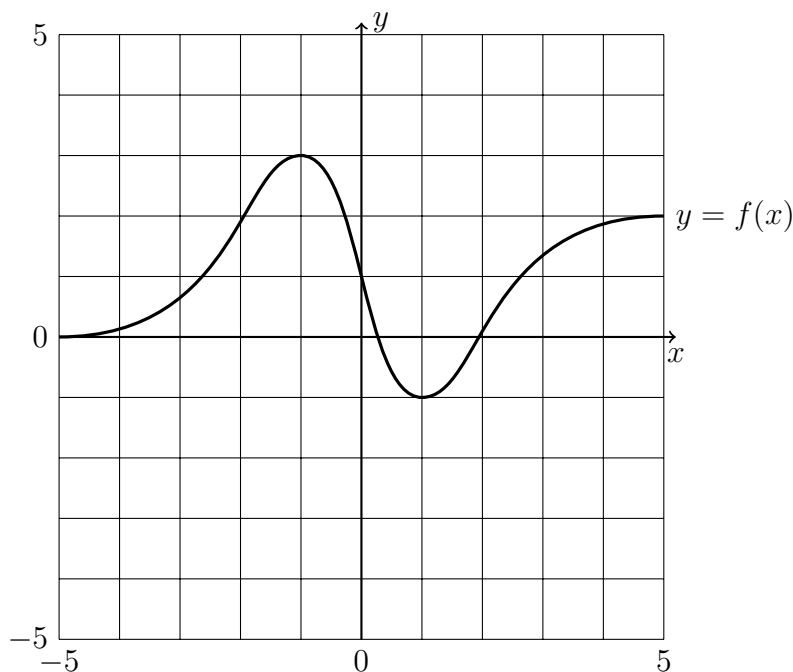
(d) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+16}-4}$

- (3) For what value of c (if any) is the function $f(x)$ continuous at $x = -1$? Justify your answer.

$$f(x) = \begin{cases} \frac{1}{x-3} - 2x & x < -1 \\ c & x = 2 \\ \frac{\cos(\pi x)}{x} & x > -1 \end{cases}$$

- (4) For a sphere of radius r , its volume is $V = \frac{4}{3}\pi r^3$. What is the average rate of change of volume when the radius increases from $r = 2$ to $r = 3$?
- (5) Show that $e^x = 4 - x^2$ has a solution for some $x > 0$. You do not need to find this solution.

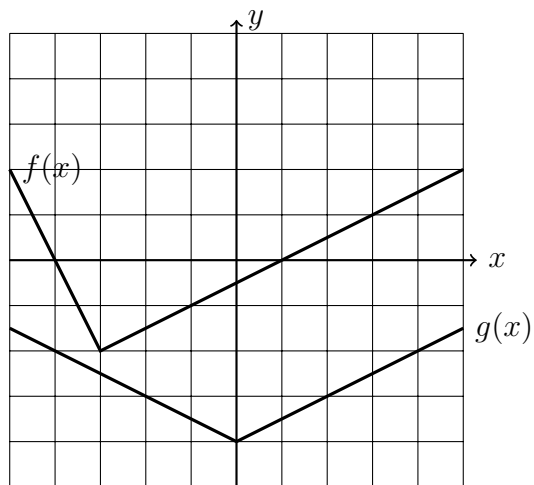
(6) Consider the function $f(x)$ defined by the following graph.



- (a) Label all regions where $f'(x) < 0$.
 - (b) Label all regions where $f'(x) > 0$.
 - (c) Sketch a graph of $f'(x)$ on the figure.
 - (d) What is $\lim_{x \rightarrow \infty} f(x)$?
 - (e) What is $\lim_{x \rightarrow -\infty} f'(x)$?
- (7) Use the limit definition of the derivative to evaluate $f'(2)$, where
- (a) $f(x) = \frac{1}{x-3}$
 - (b) $f(x) = \frac{1}{\sqrt{1+x}}$
- (8) Find the derivatives of the following functions

- | | |
|---|--|
| (a) $2x^4 - 3x^2 + \sqrt{x^5} - 3\sqrt[5]{1/x^2}$ | (f) $\sin^2(x)$ |
| (b) $4x^3 e^x$ | (g) $x^4 e^x \cos(x)$ |
| (c) $\frac{5x+2}{4-3x}$ | (h) $\frac{\sqrt{x+e^x}}{1+\cos^{-1}(3x)}$ |
| (d) $\frac{5-\sqrt{2x}}{2-\cos(x)}$ | (i) x^{x^2} |
| (e) $\tan(x)$ | (j) $\sqrt{\ln(\sec(x))}$ |
| | (k) $\cos^{-1}(3x-2)$ |

- (9) Find the second derivatives of the functions above (skip (d, j, h, k) as they takes a long time).
- (10) The graphs of the functions f and g are shown below.



- (a) Let $h(x) = f(x)g(x)$. Find $h'(3)$.
- (b) Let $h(x) = f(x)/g(x)$. Find $h'(-1)$.
- (c) Let $h(x) = f(g(x))$. Find $h'(2)$.