

Math 231 Calculus 1 Spring 22 Midterm 1b

Name: Solutions

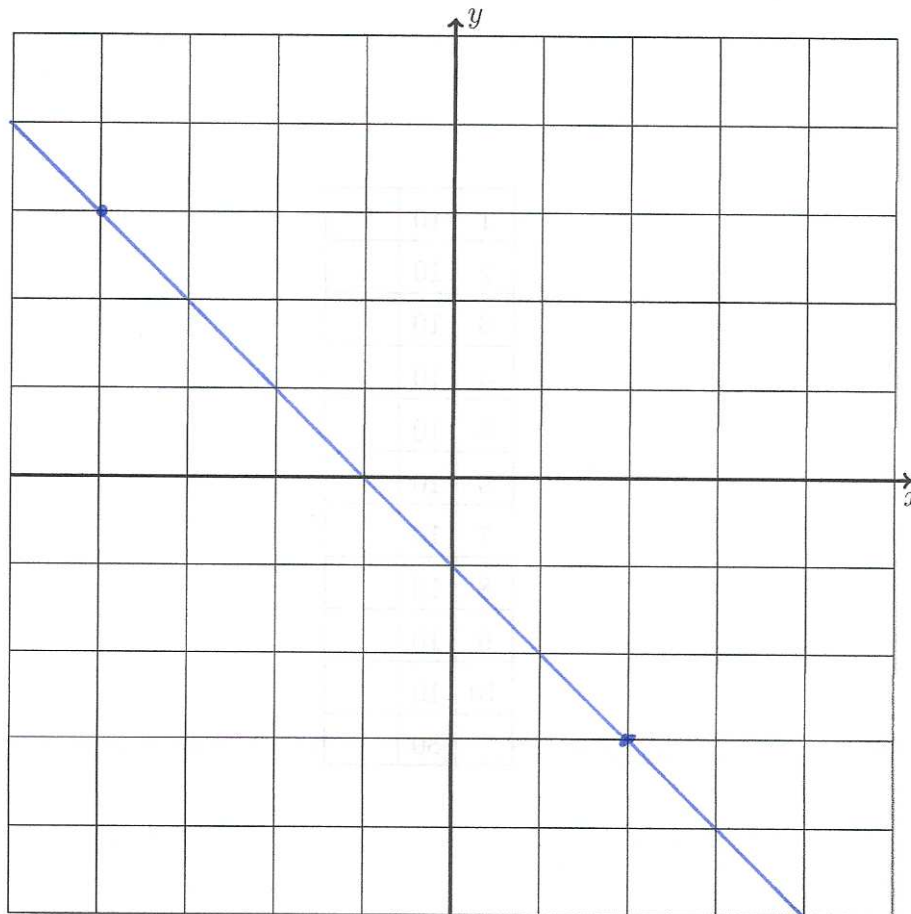
- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

- (1) (10 points) Plot the points $(2, -3)$ and $(-4, 3)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

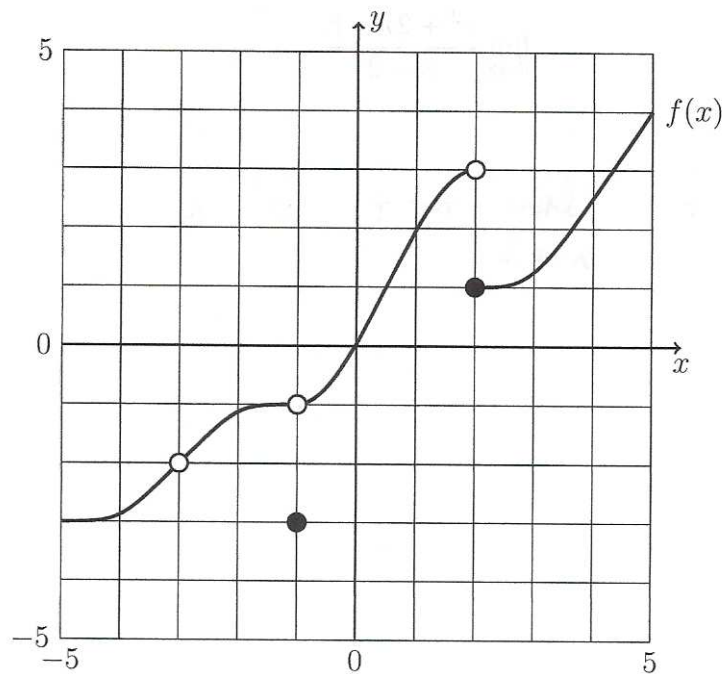
$$\text{slope } m = \frac{3 - (-3)}{-4 - 2} = \frac{6}{-6} = -1$$



$$y - 3 = -1(x - (-4))$$

$$y = -x - 1$$

- (2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



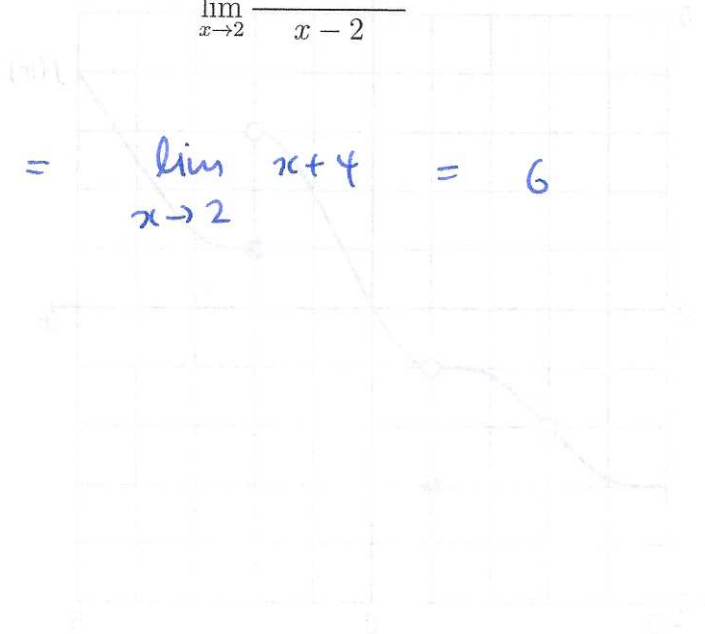
- (a) $\lim_{x \rightarrow -3^+} f(x)$ -2
 (b) $\lim_{x \rightarrow -3} f(x)$ -2
 (c) $\lim_{x \rightarrow -1^-} f(x)$ -1
 (d) $\lim_{x \rightarrow -1} f(x)$ -1
 (e) $\lim_{x \rightarrow 2^+} f(x)$ 1
 (f) $\lim_{x \rightarrow 2} f(x)$ DNE

- (3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x - 2}$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+4)}{(x-2)}$$

$$= \lim_{x \rightarrow 2} x + 4 = 6$$



- (4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \rightarrow 9} \frac{x - 9}{3 - \sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{(\sqrt{x} - 3)(\sqrt{x} + 3)}{3 - \sqrt{x}} = \lim_{x \rightarrow 9} -(\sqrt{x} + 3) = -6$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 2x$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h - 2h = 2x - 2$$

(6) (10 points) Find the following limit.

$$\lim_{x \rightarrow \infty} \frac{2x - 2}{\sqrt{3x^2 - 3}}$$

$$\lim_{x \rightarrow \infty} \frac{2 - 2/x}{\sqrt{3 - 3/x^2}} = \frac{2}{\sqrt{3}}$$

$$-2x^{-1/2}$$

(7) Find the first and second derivatives of $f(x) = x^3 - \cos(x) - 2/\sqrt{x}$.

$$f'(x) = 3x^2 + \sin(x) + x^{-3/2}$$

$$f''(x) = 6x + \cos(x) - \frac{3}{2}x^{-5/2}$$

(8) Find the first and second derivatives of $f(x) = \frac{x}{e^x} - \sqrt[3]{x}$.

$$f'(x) = \frac{e^x \cdot 1 - e^x \cdot x}{e^{2x}} - \frac{1}{3} x^{-2/3} = -\frac{x}{e^x} + e^{-x} - \frac{1}{3} x^{-2/3}$$

$$f''(x) = -\frac{x}{e^x} + e^{-x} + e^{-x} + \frac{2}{9} x^{-5/3} = 2e^{-x} - \frac{x}{e^x} + \frac{2}{9} x^{-5/3}$$

(9) Find the first and second derivatives of $f(x) = \sin(x^2)$.

$$f'(x) = \cos(x^2) \cdot 2x$$

$$f''(x) = -\sin(x^2) \cdot 4x^2 + 2\cos(x^2)$$

- (10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.

