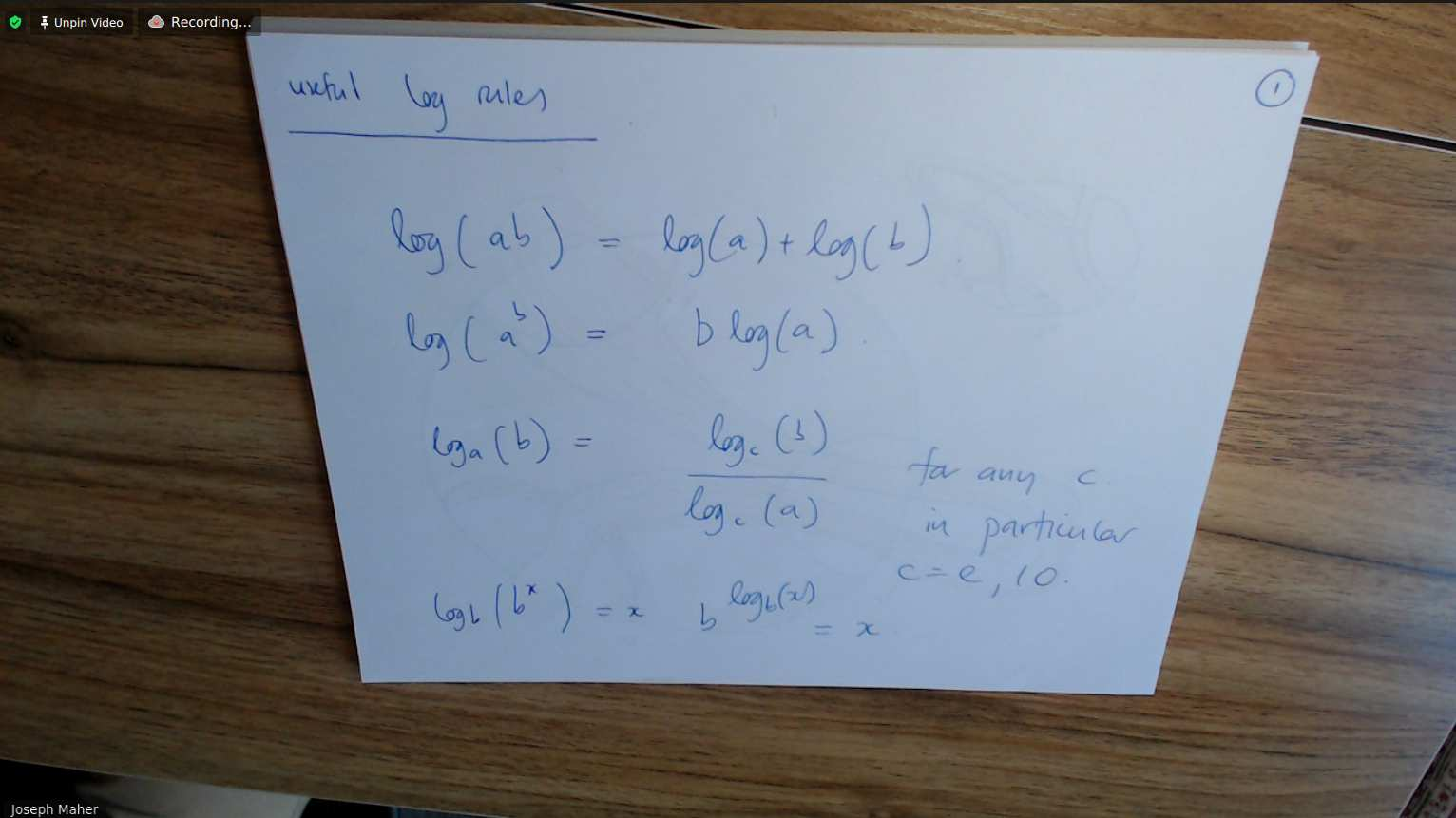


## useful log rules

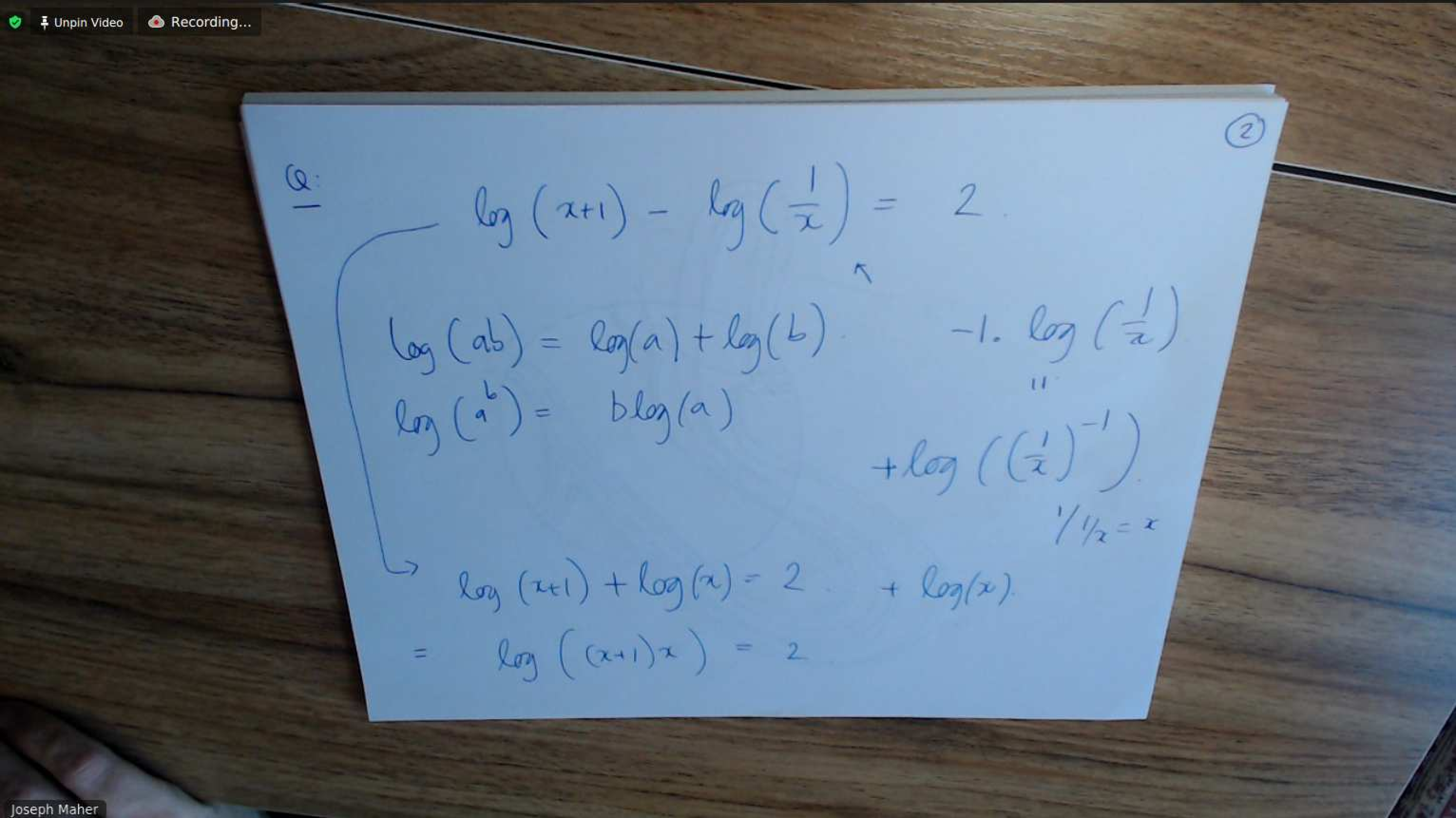
$$\log(ab) = \log(a) + \log(b)$$

$$\log(a^b) = b\log(a)$$

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

for any c
in particular
$c = e, 10.$

$$\log_b(b^x) = x \qquad b^{\log_b(x)} = x$$



②

**Q:** $\log(x+1) - \log\left(\frac{1}{x}\right) = 2$

↖

$\log(ab) = \log(a) + \log(b)$

$\log(a^b) = b\log(a)$

$-1.\log\left(\frac{1}{x}\right)$

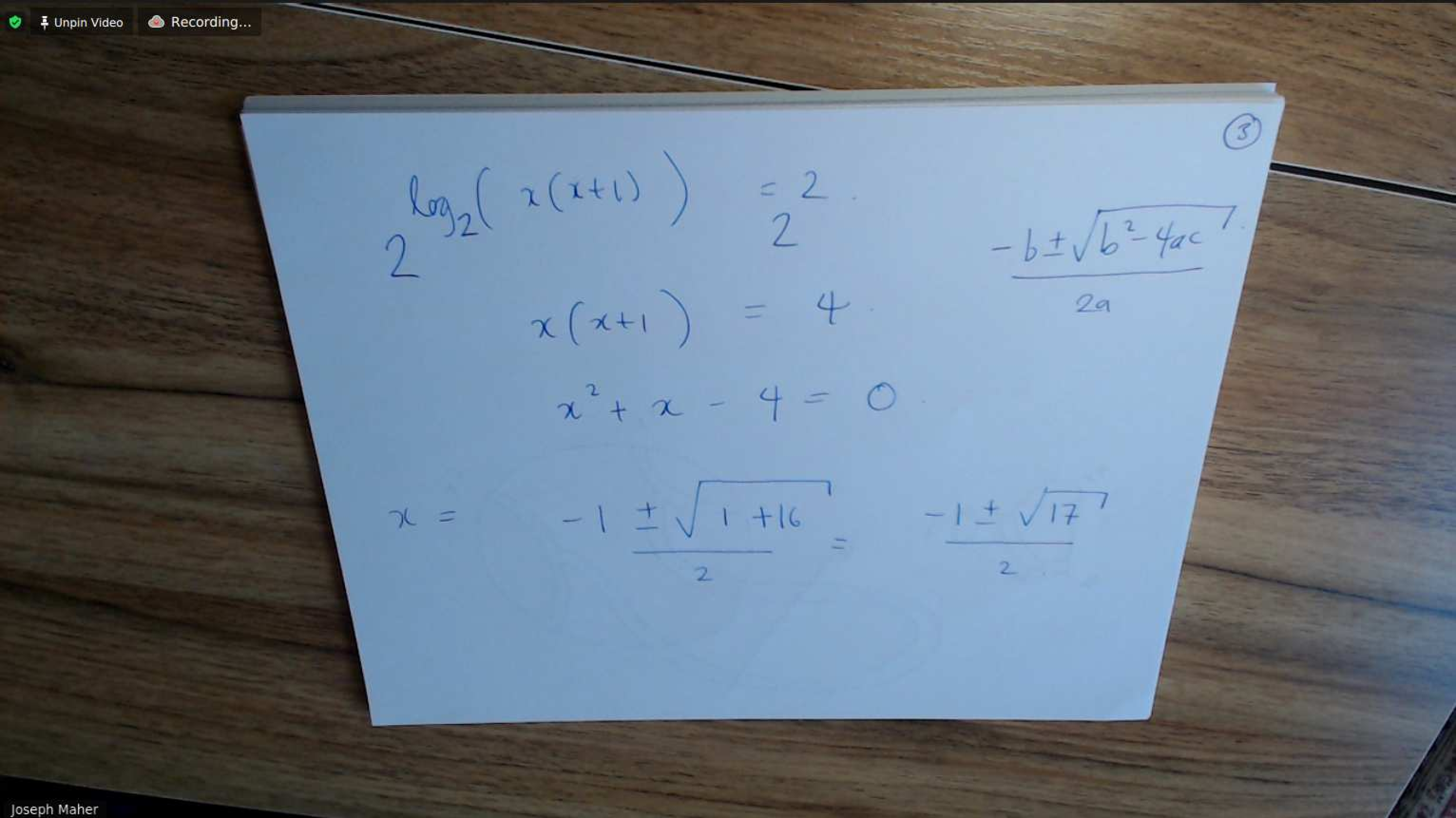
''

$+\log\left(\left(\frac{1}{x}\right)^{-1}\right)$

$1/{}^{1}/_{x} = x$

$\log(x+1) + \log(x) = 2$ $\quad + \log(x)$

$= \log((x+1)x) = 2$



③

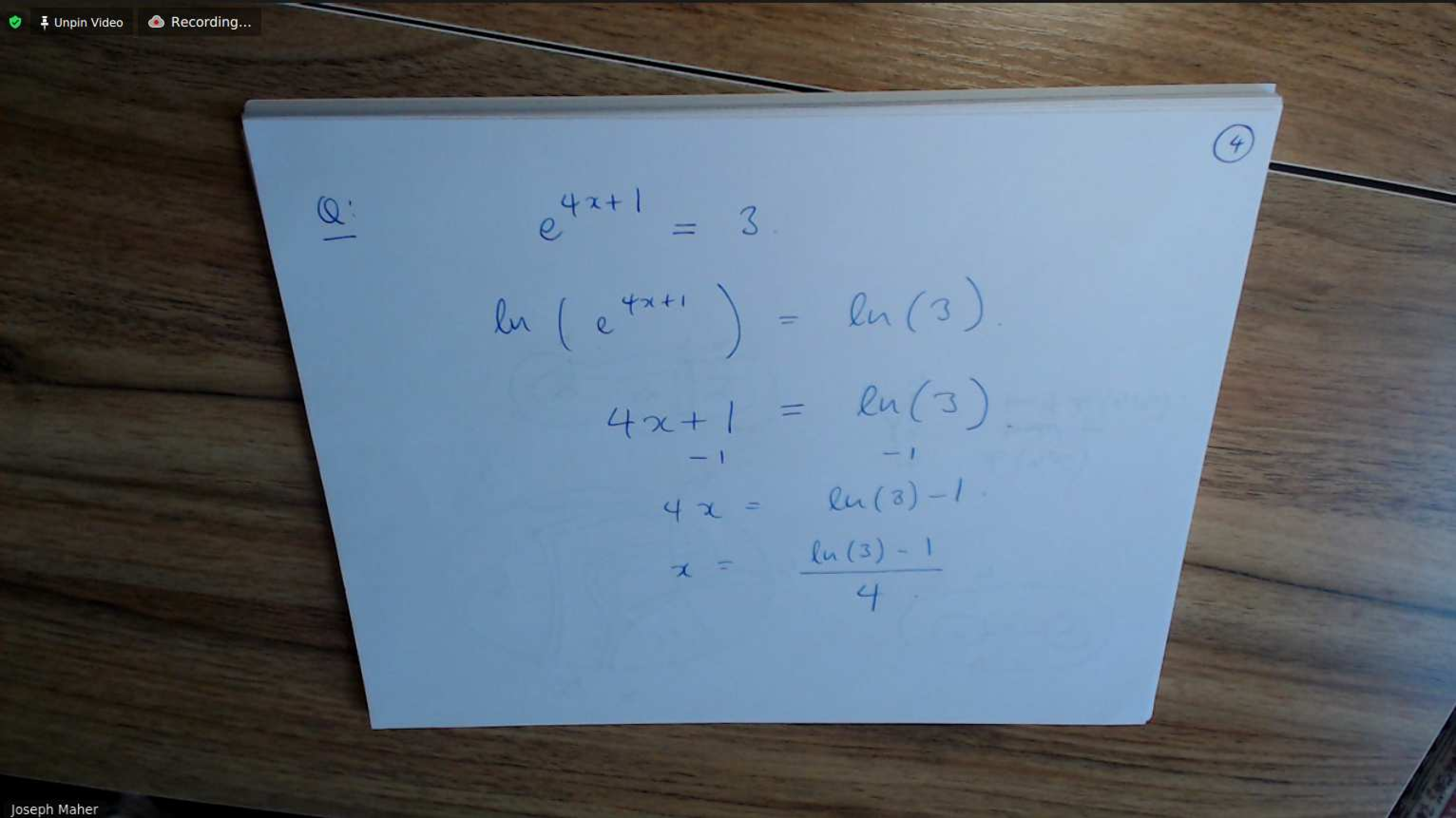
$$2^{\log_2( x(x+1) )} = 2^2$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x(x+1) = 4$$

$$x^2 + x - 4 = 0$$

$$x = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$



④

**Q:**

$$e^{4x+1} = 3.$$

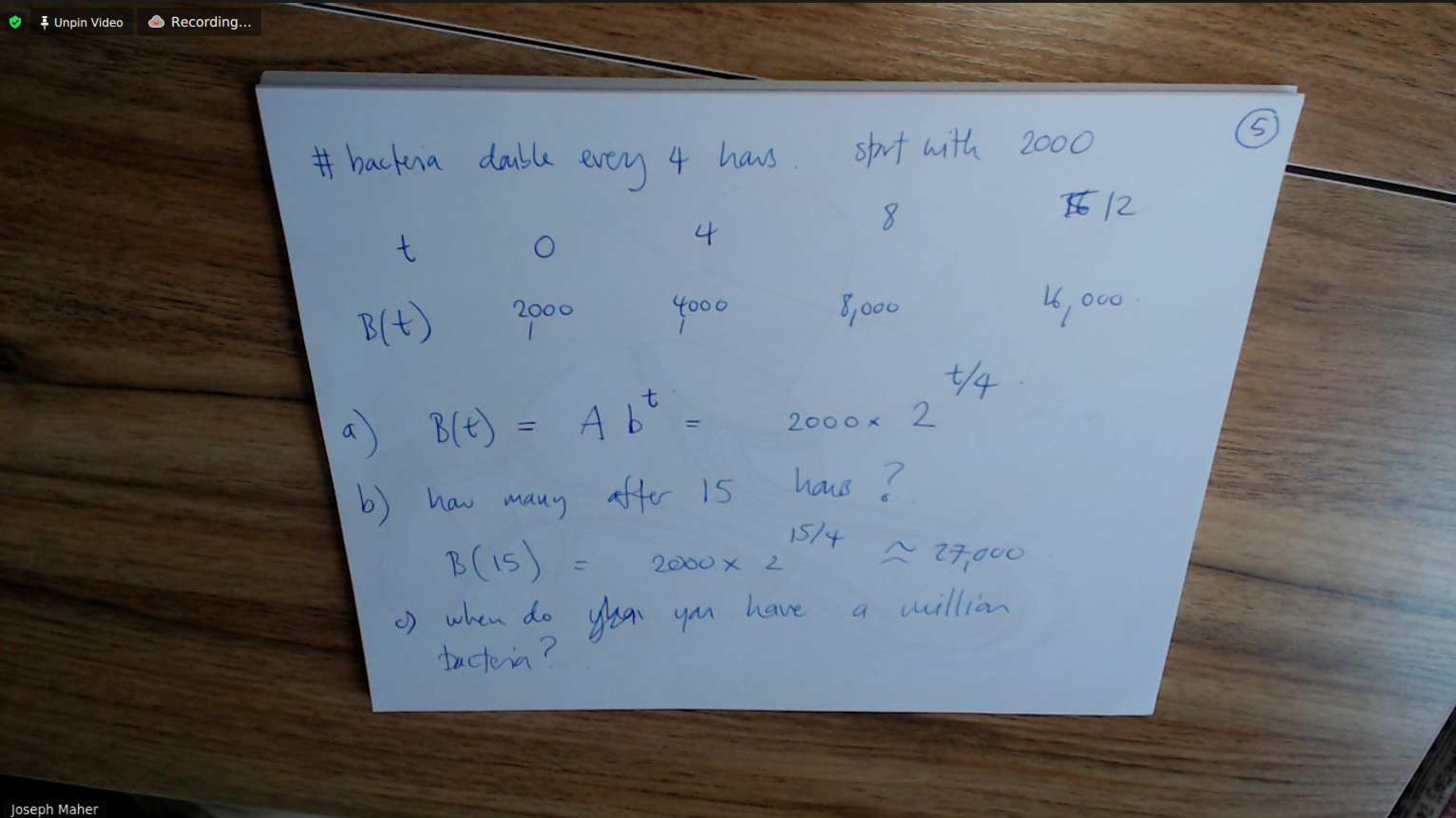
$$\ln\left(e^{4x+1}\right) = \ln(3).$$

$$4x+1 = \ln(3)$$

$$-1 \qquad -1$$

$$4x = \ln(3)-1.$$

$$x = \frac{\ln(3)-1}{4}.$$



# bacteria double every 4 hours. start with 2000

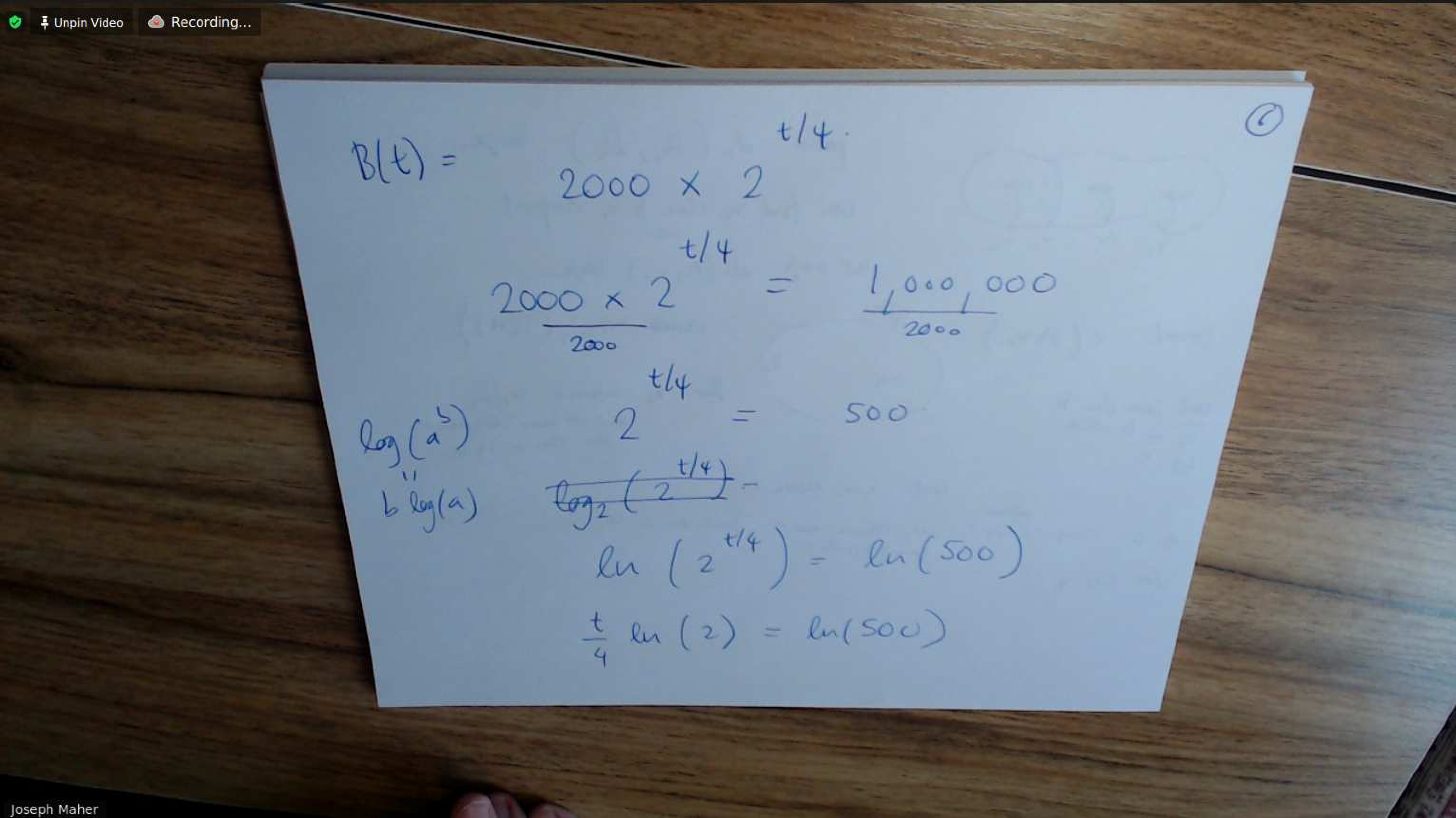
| | | | | |
|---|---|---|---|---|
| $t$ | 0 | 4 | 8 | ~~16~~ 12 |
| $B(t)$ | 2000 | 4000 | 8,000 | 16,000 |

a) $B(t) = A\,b^t = 2000 \times 2^{t/4}$.

b) how many after 15 hours?

$$B(15) = 2000 \times 2^{15/4} \simeq 27{,}000$$

c) when do ~~you~~ you have a million bacteria?



$$B(t) = 2000 \times 2^{t/4}.$$

$$\frac{2000 \times 2^{t/4}}{2000} = \frac{1,000,000}{2000}$$

$$2^{t/4} = 500$$

$\log(a^b)$

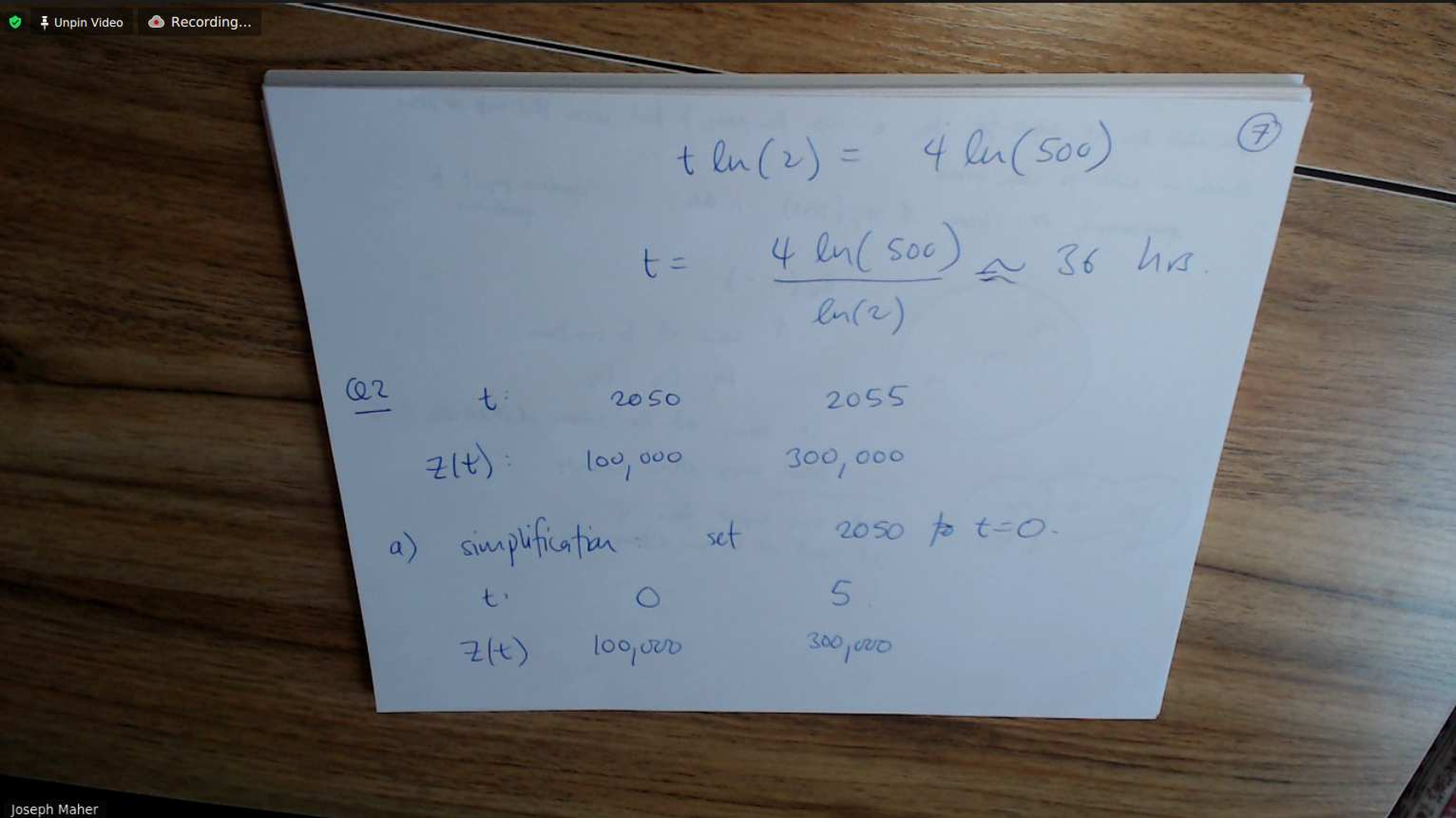
$''$

$b\log(a)$

~~$\log_2(2^{t/4})$~~ $=$

$$\ln(2^{t/4}) = \ln(500)$$

$$\frac{t}{4}\ln(2) = \ln(500)$$



$$t \ln(2) = 4 \ln(500)$$

$$t = \frac{4 \ln(500)}{\ln(2)} \approx 36 \text{ hrs.}$$

**Q2**

| | | |
|---|---|---|
| t: | 2050 | 2055 |
| Z(t): | 100,000 | 300,000 |

a) simplification: set 2050 to $t=0$.

| | | |
|---|---|---|
| t: | 0 | 5 |
| Z(t) | 100,000 | 300,000 |

a)

| | | |
|---|---|---|
| $t$ | $0$ | $5$ |
| $z(t)$ | $100,000$ | $300,000$ |

$$z(t) = Ab^t = \boxed{Ae^{ct}}$$

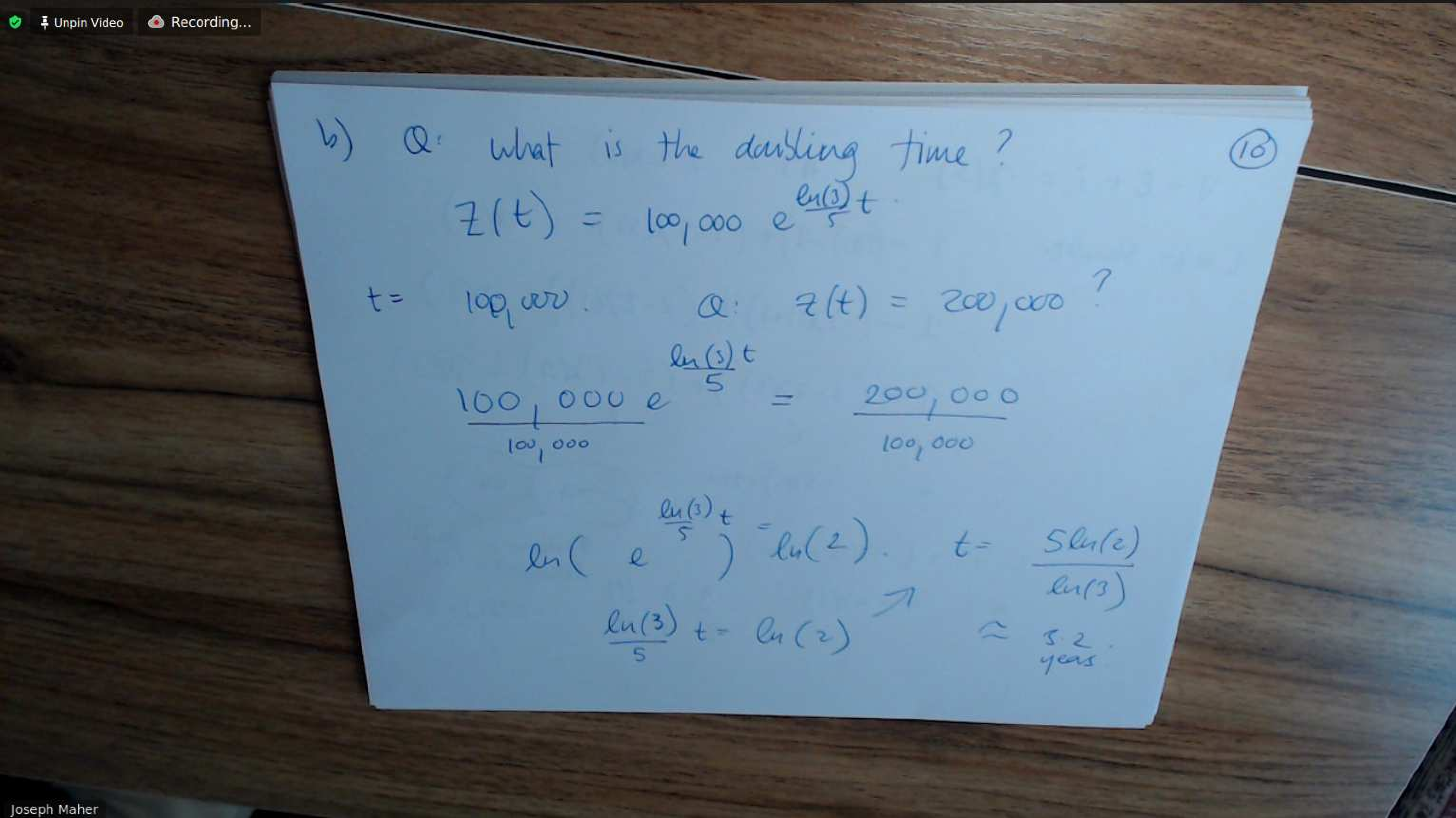
$$b = e^{\ln(b)}$$

$$b^t = e^{\ln(b)t}$$

$t=0$ $\quad A = 100,000$

$t=5$ $\quad \underbrace{100,000}_{z(t)} \neq A \quad 100,000\, e^{c5} = z(5) = 300,000$

$$100,000\, e^{5c} = 300,000$$



b) Q: what is the doubling time? ⑩

$$Z(t) = 100,000\, e^{\frac{\ln(3)}{5}t}$$

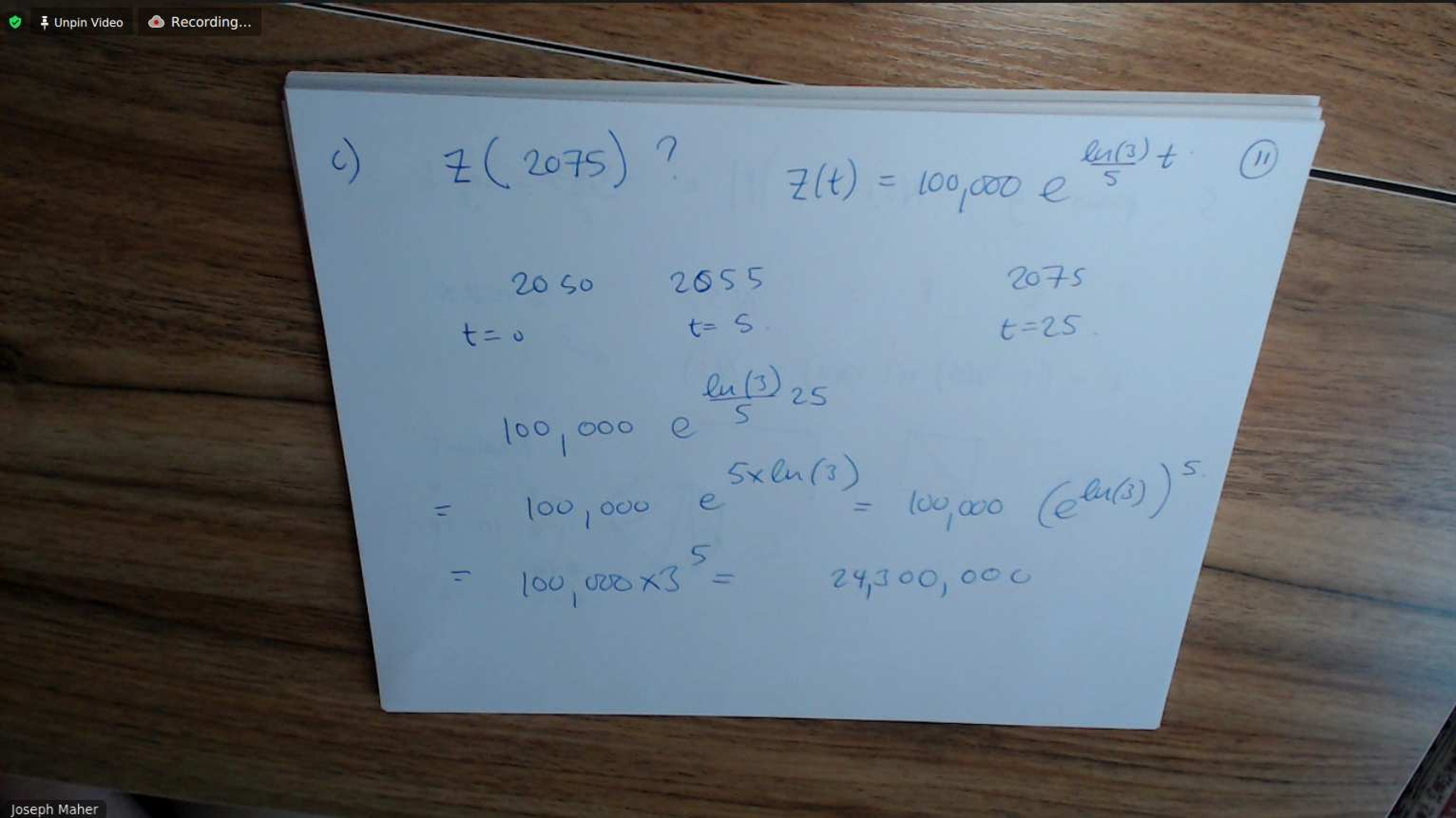
t = 100,000 .   Q: $z(t) = 200,000$ ?

$$\frac{100,000}{100,000}\, e^{\frac{\ln(3)}{5}t} = \frac{200,000}{100,000}$$

$$\ln\left(e^{\frac{\ln(3)}{5}t}\right) = \ln(2).$$

$$\frac{\ln(3)}{5}t = \ln(2) \nearrow \quad t = \frac{5\ln(2)}{\ln(3)}$$

$$\approx 3.2 \text{ years}$$



c) $Z(2075)$ ?  $Z(t) = 100{,}000\, e^{\frac{\ln(3)}{5} t}$

| | | |
|---|---|---|
| 2050 | 2055 | 2075 |
| $t=0$ | $t=5$ | $t=25$ |

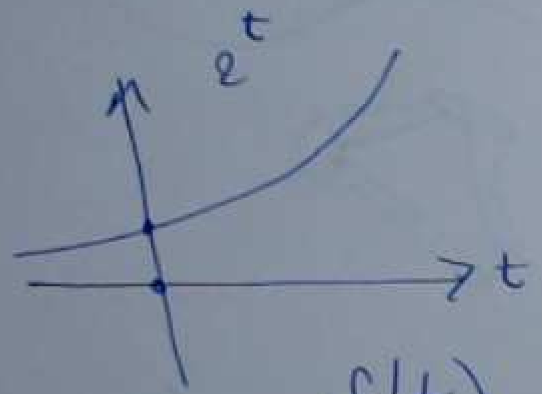
$$100{,}000\, e^{\frac{\ln(3)}{5} 25}$$

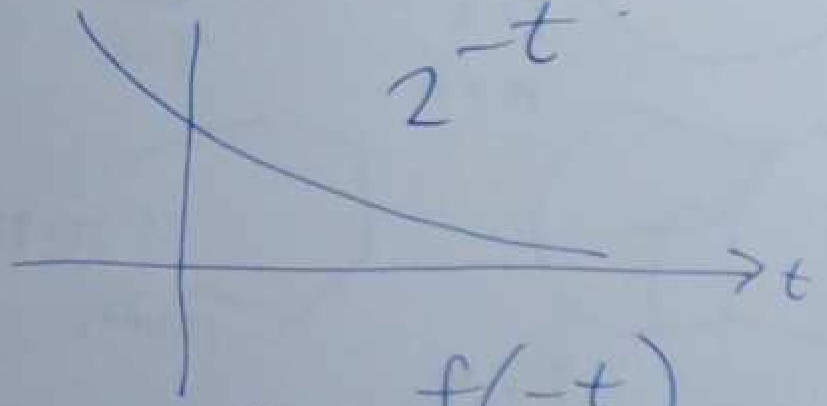
$$= 100{,}000\, e^{5 \times \ln(3)} = 100{,}000\, \left(e^{\ln(3)}\right)^{5}$$

$$= 100{,}000 \times 3^{5} = 24{,}300{,}000$$

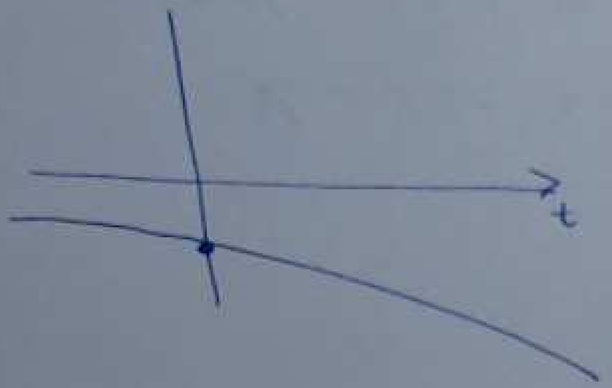
half life of 28 years. $t=0$ 50 mg

Q: how long till we have 32 mg?



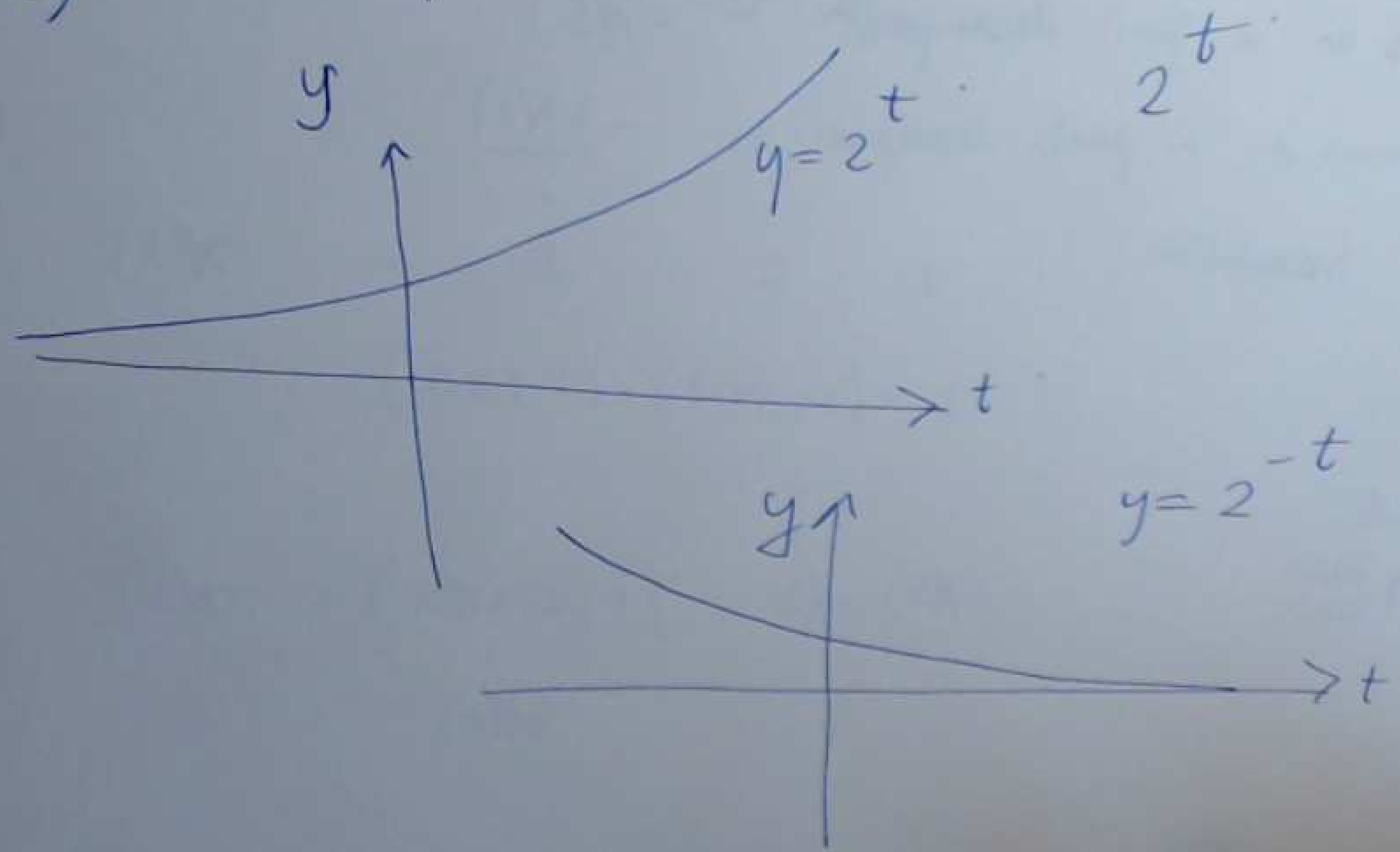


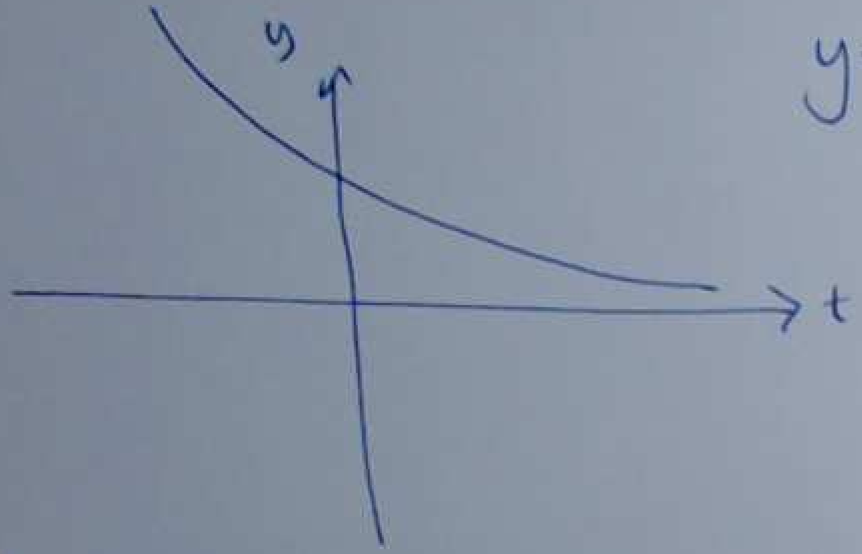
$$f(t) \xrightarrow[\text{reflection in the vertical axis}]{} f(-t)$$



$$-2^t = -(2^t)$$

$$f(t) = 4 - 2^{-t}$$

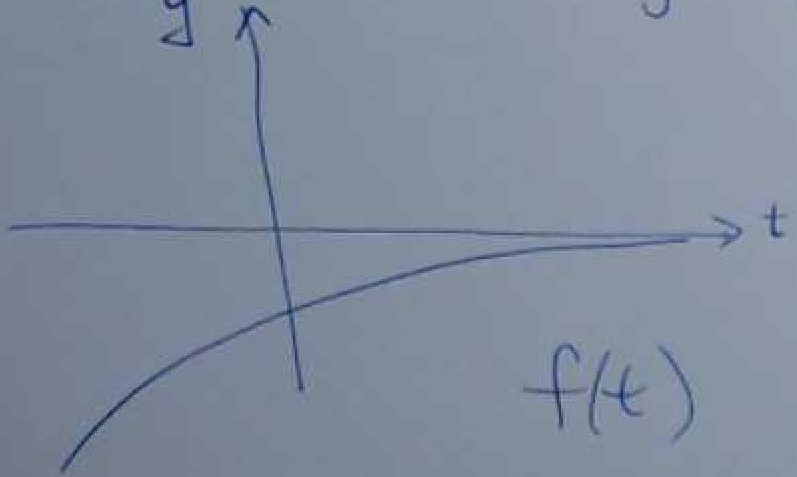




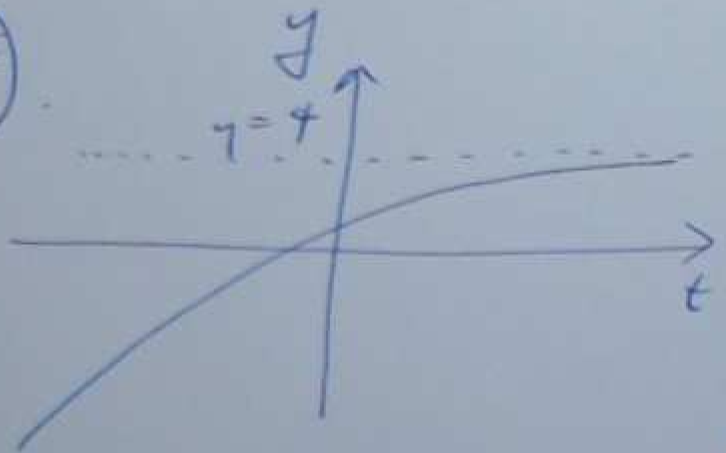
$y = 2^{-t}$

$y = 4 - 2^{-t}$

$y = -(2^{-t})$



$f(t)$



$f(t) + 4$

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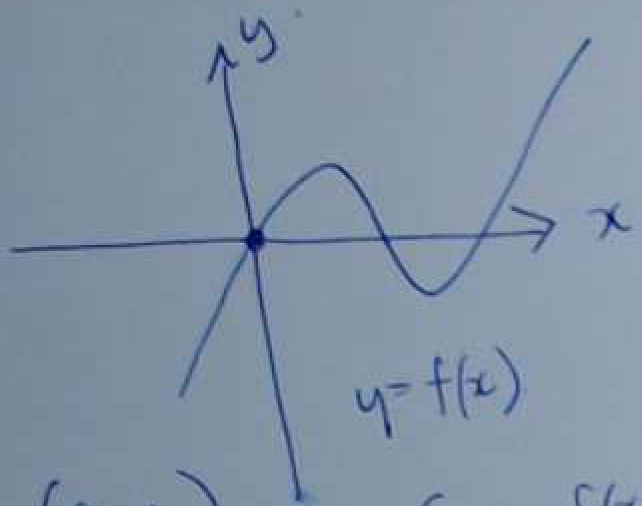
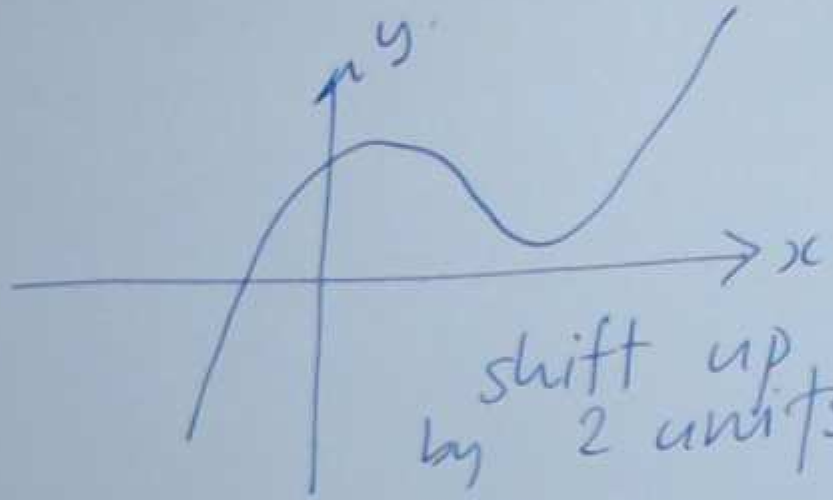


(0, 0)

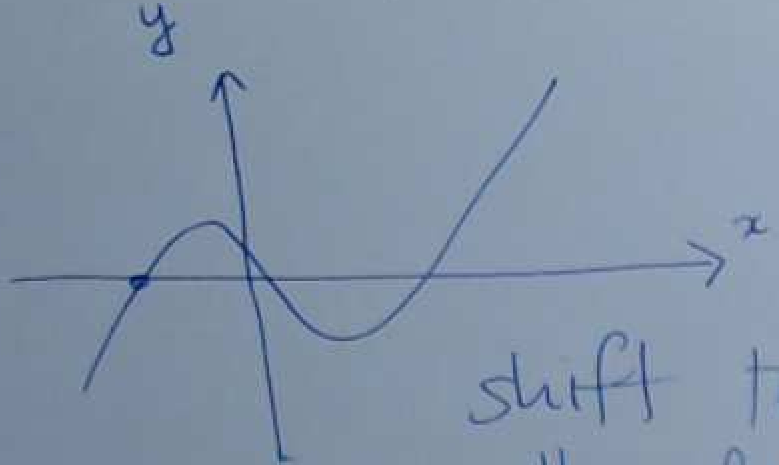
(x, f(x))



shift up by 2 units.

$y = f(x) + 2$.

$(x, f(x) + 2)$.



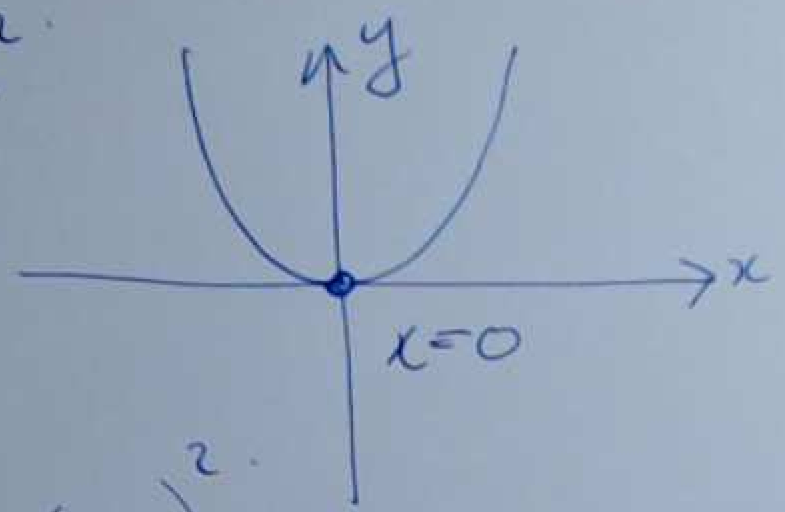
shift to the left

$y = f(x+2)$

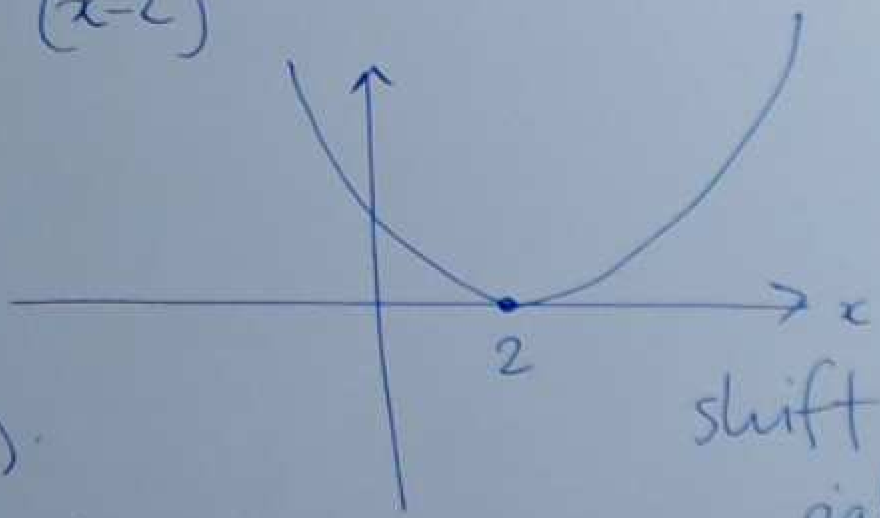
$(-2, 0)$.

Joseph Maher

$f(x) = x^2$.

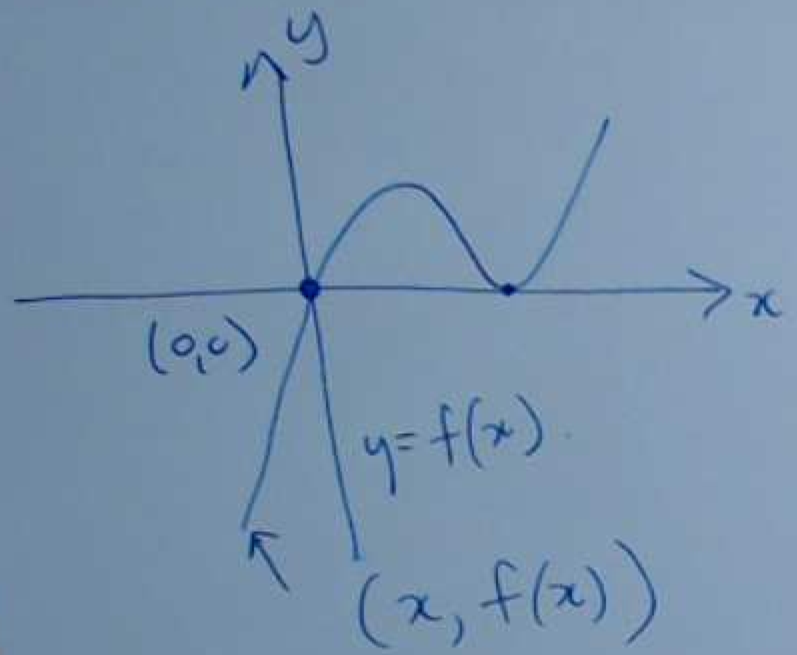


$f(x-2) = (x-2)^2$.



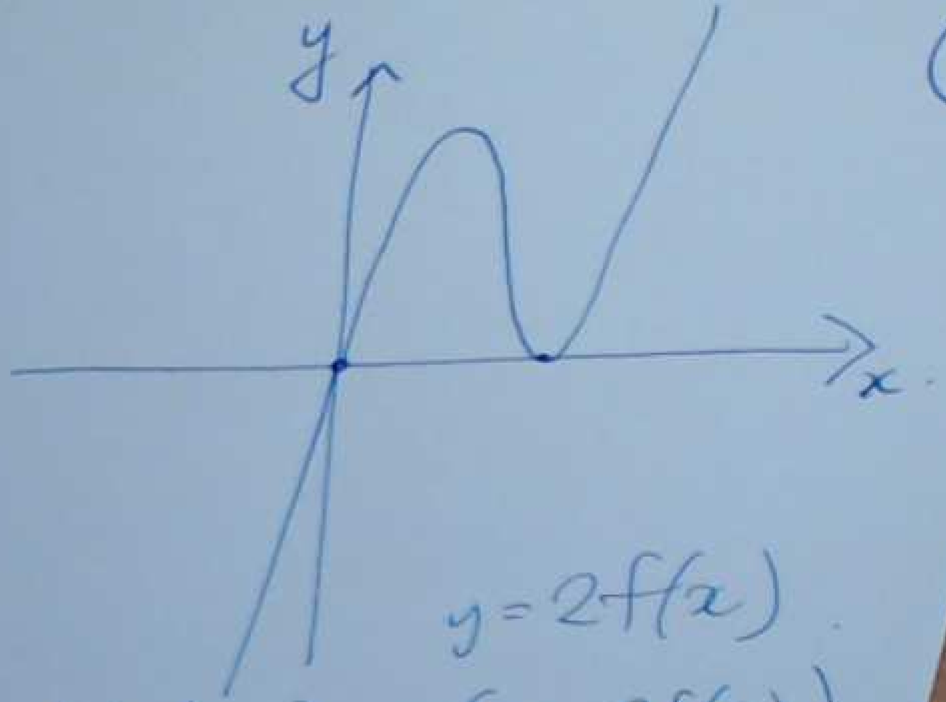
$f(x)$.
$f(x-2)$

shift to the right.



y = f(x)

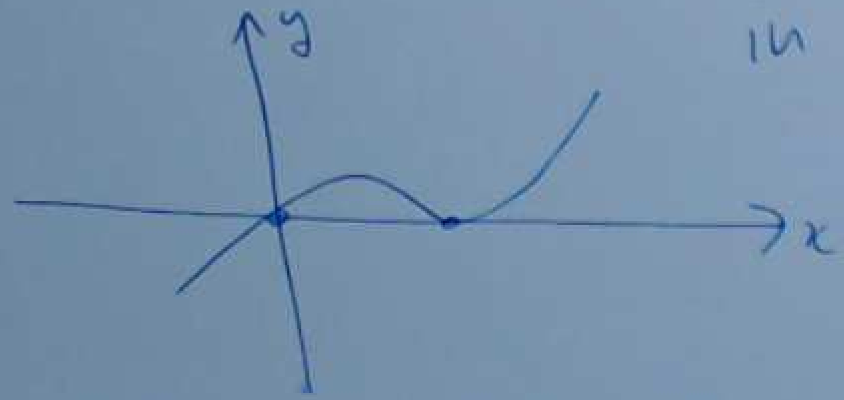
(x, f(x))



y = 2f(x)

(x, 2f(x))

stretch by 2
in y-direction



$y = \frac{1}{2} f(x)$

shrink in y-dir.

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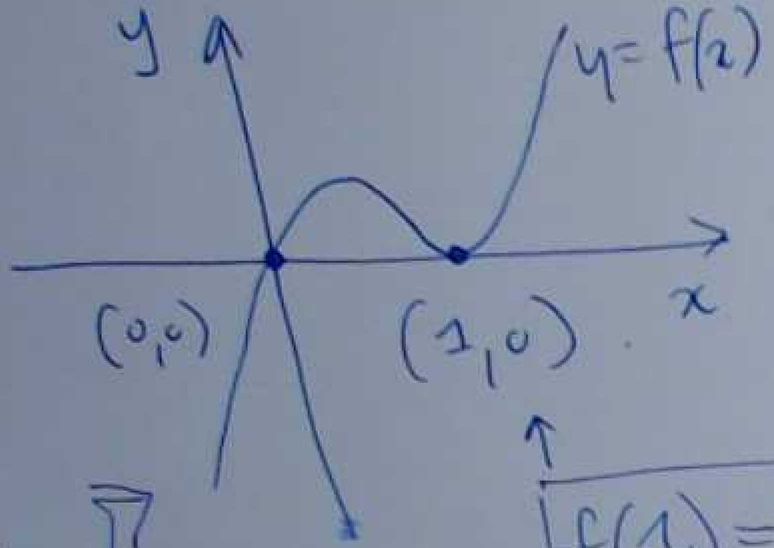
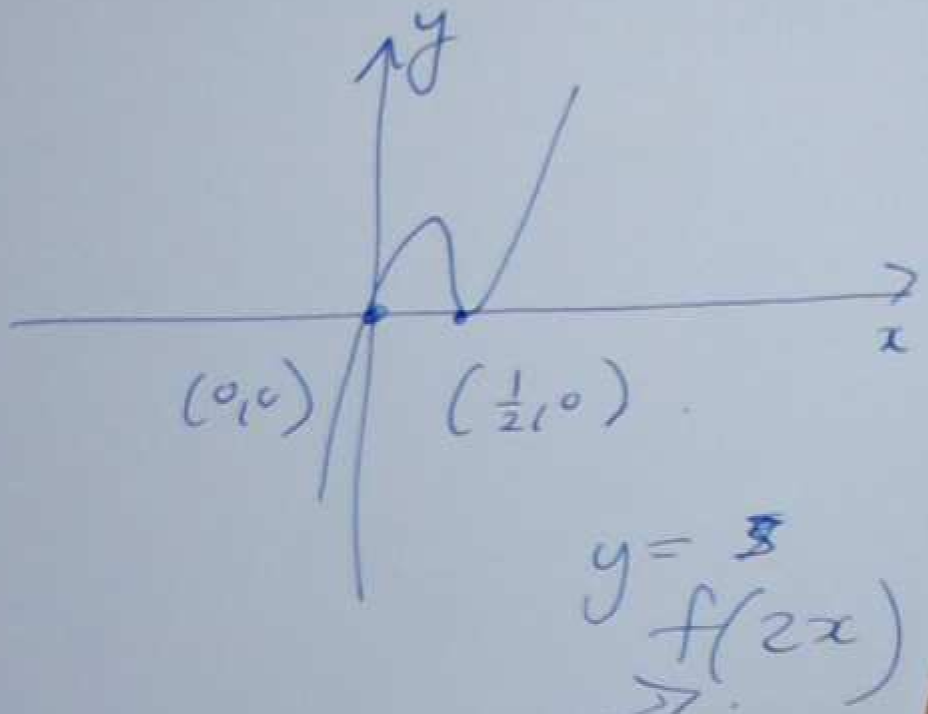


$\boxed{f(1)=0}$



$f(2\times2) = f(4)$.

$y = f(2x)$

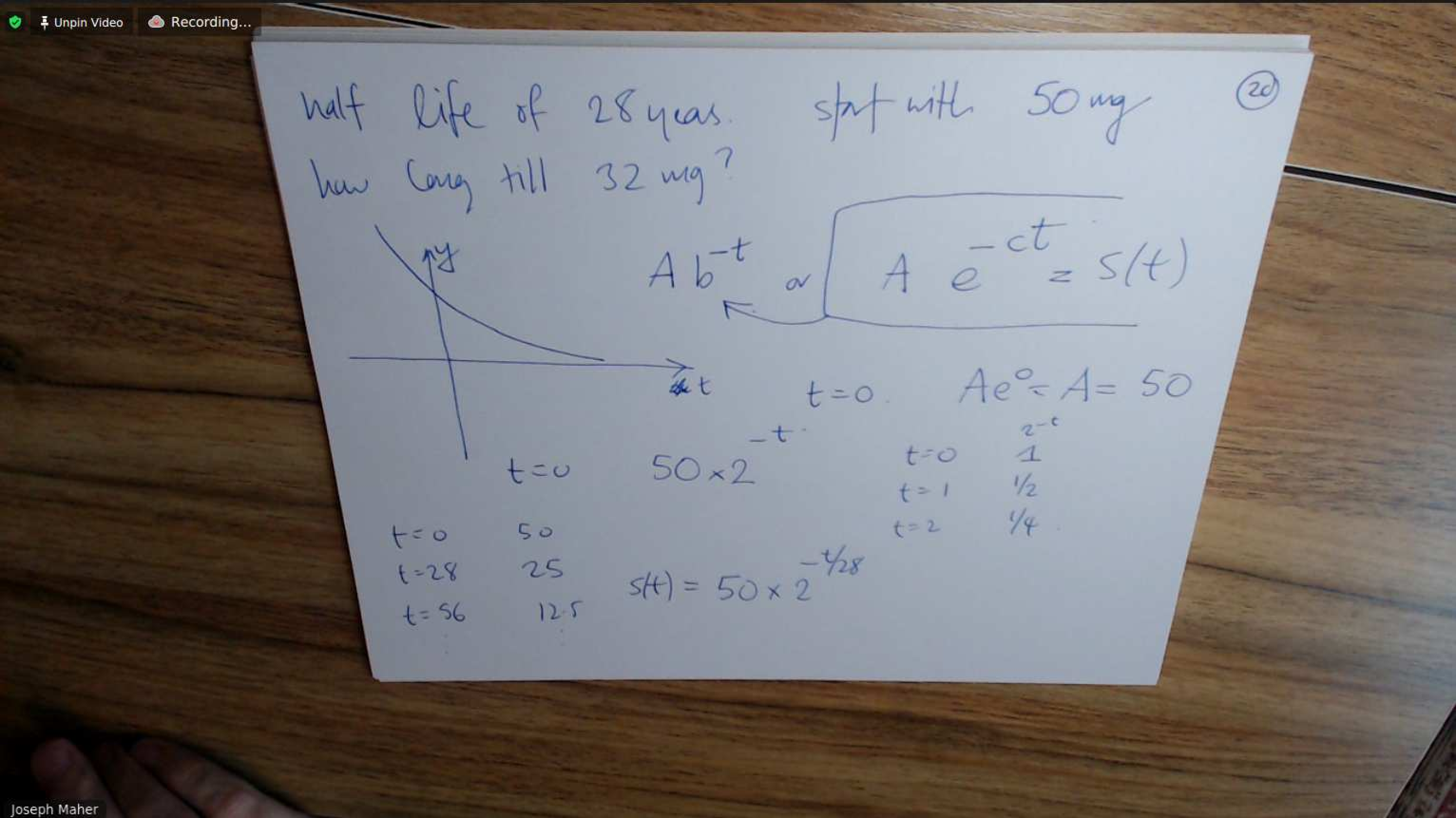
shrink in horizontal direction.

$f(2\times\frac{1}{2}) = f(1) = 0$

$(x, f(x)) \rightarrow (x, f(2x))$ " $(\frac{1}{2}, f(1))$.

$(0,0)$ $(1,0)$ $(\frac{1}{2}, f(2\times\frac{1}{2}))$ " $(\frac{1}{2},0)$

Joseph Maher



half life of 28 years. start with 50 mg ⑳

how long till 32 mg?

$A b^{-t}$ or $\boxed{A e^{-ct} = S(t)}$

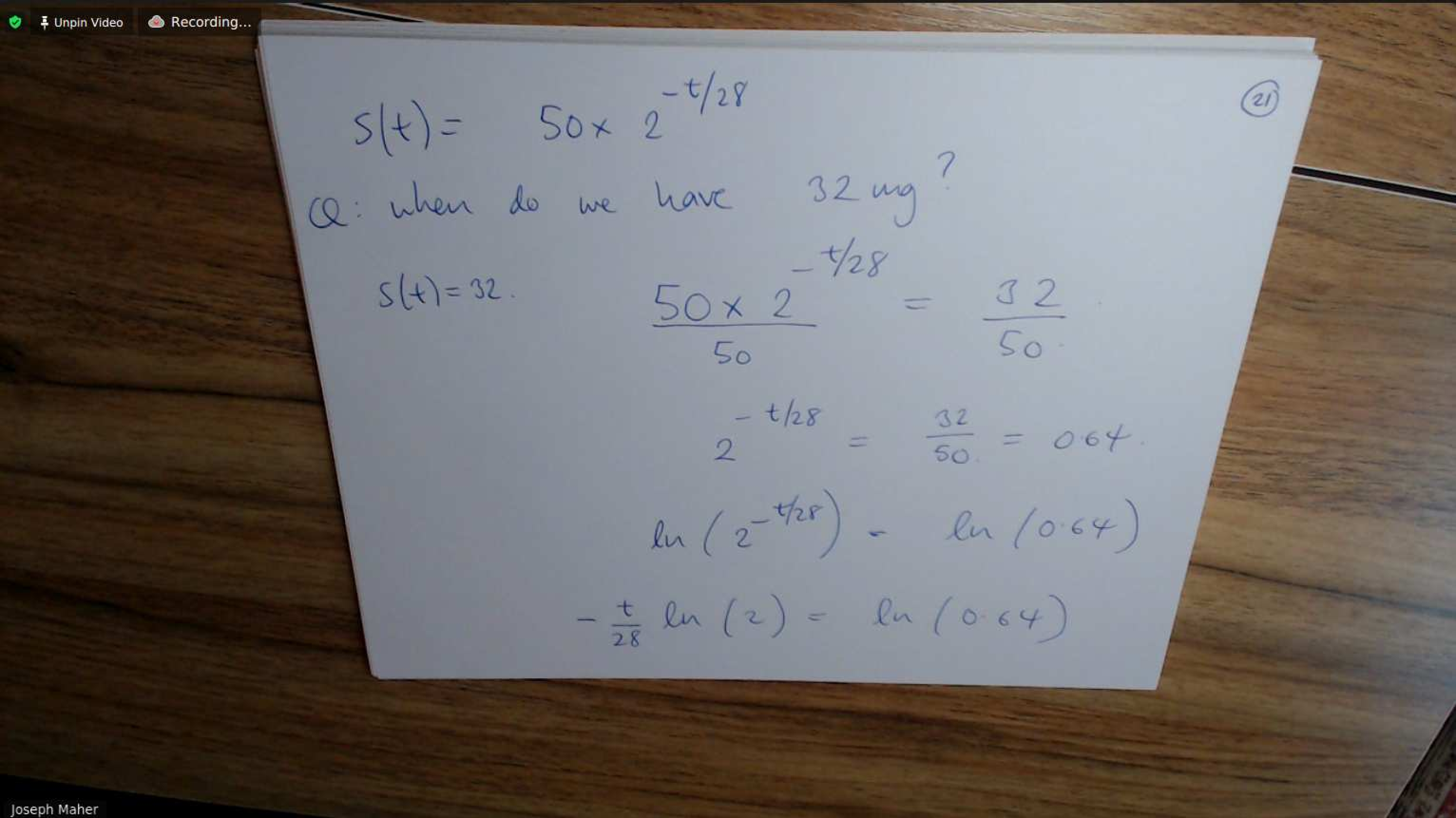
t=0 $Ae^0 = A = 50$

t=0 $50 \times 2^{-t}$

|  | $2^{-t}$ |
|---|---|
| t=0 | 1 |
| t=1 | ½ |
| t=2 | ¼ |

|  |  |
|---|---|
| t=0 | 50 |
| t=28 | 25 |
| t=56 | 12.5 |

$s(t) = 50 \times 2^{-t/28}$



㉑

$s(t) = 50 \times 2^{-t/28}$

Q: when do we have 32 mg?

$s(t) = 32.$

$$\frac{50 \times 2^{-t/28}}{50} = \frac{32}{50}$$

$$2^{-t/28} = \frac{32}{50} = 0.64$$

$$\ln\left(2^{-t/28}\right) - \ln(0.64)$$

$$-\frac{t}{28}\ln(2) = \ln(0.64)$$