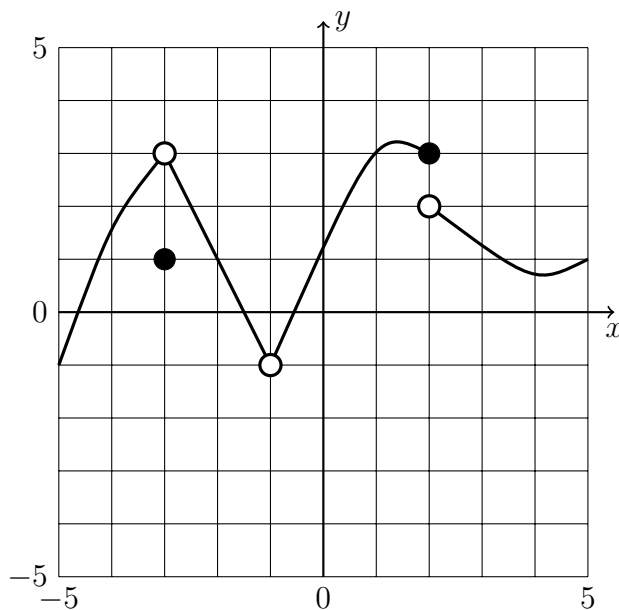


Math 231 Calculus 1 Fall 21 Sample Midterm 1

- (1) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



- (a) $\lim_{x \rightarrow 2^-} f(x)$
(b) $\lim_{x \rightarrow 2^+} f(x)$
(c) $\lim_{x \rightarrow 2} f(x)$
(d) $\lim_{x \rightarrow -1^-} f(x)$
(e) $\lim_{x \rightarrow -1^+} f(x)$
(f) $\lim_{x \rightarrow -3} f(x)$

- (2) Evaluate these limits. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

(a) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x + 2}$

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{3 + 2x^2}}{2 - x}$

(c) $\lim_{x \rightarrow 0} \frac{\sin 4x}{5x}$

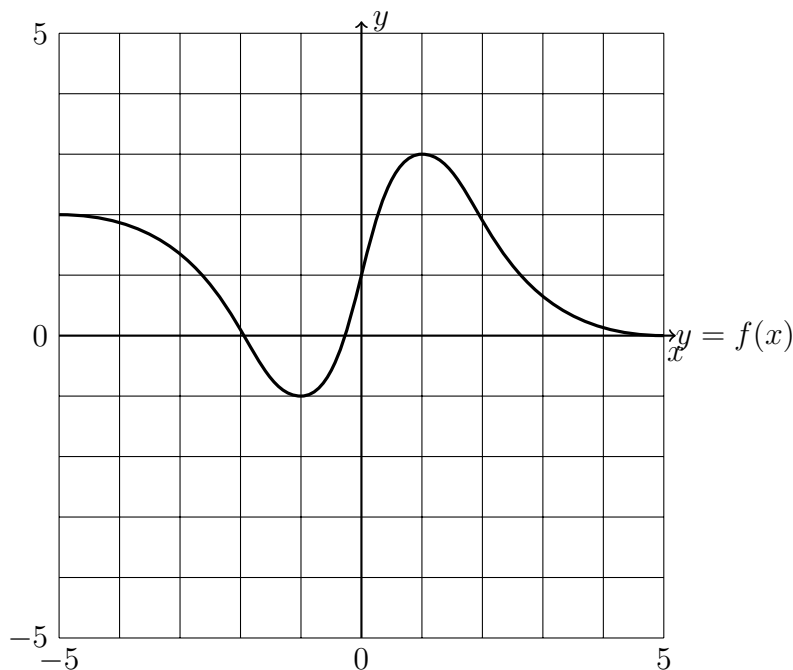
(d) $\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+1}} \right)$

- (3) For what value of c (if any) is the function $f(x)$ continuous at $x = 2$? Justify your answer.

$$f(x) = \begin{cases} 2x - \frac{1}{x-2} & x < 3 \\ c & x = 3 \\ \frac{\cos(\pi x)}{x} & x > 3 \end{cases}$$

- (4) For a sphere of radius r , its volume is $V = 4/3\pi r^3$. What is the average rate of change of volume when the radius increases from $r = 2$ to $r = 4$?
- (5) Show that $e^x = 1/x$ has a solution for some $x > 0$. You do not need to find this solution.

(6) Consider the function $f(x)$ defined by the following graph.

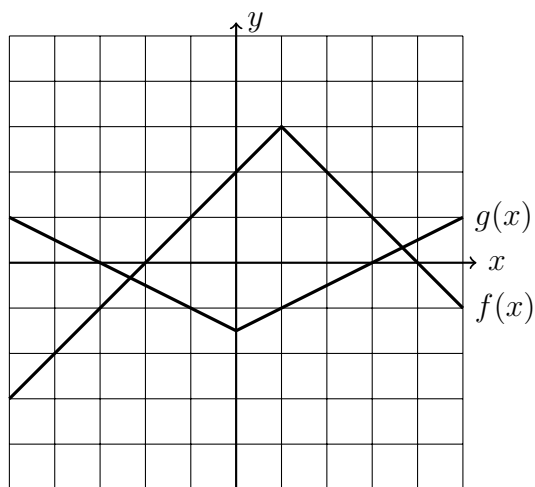


- (a) Label all regions where $f'(x) < 0$.
 - (b) Label all regions where $f'(x) > 0$.
 - (c) Sketch a graph of $f'(x)$ on the figure.
 - (d) What is $\lim_{x \rightarrow \infty} f(x)$?
 - (e) What is $\lim_{x \rightarrow -\infty} f'(x)$?
- (7) Use the limit definition of the derivative to evaluate $f'(3)$, where
- (a) $f(x) = \frac{2}{x-1}$
 - (b) $f(x) = \frac{1}{\sqrt{x+1}}$
- (8) Find the derivatives of the following functions
- (a) $2x^2 - x^3 - 2\sqrt{x} + 4\sqrt[3]{1/x^2}$
 - (b) $2x^3 e^x$
 - (c) $\frac{2x+3}{2x-3}$
 - (d) $\frac{\sqrt{2x-1}}{3-\cos(x)}$
 - (e) $\tan(x)$

(f) $\sin^2(x)$

(9) Find the second derivatives of the functions above (skip (d) as it takes a long time).

(10) The graphs of the functions f and g are shown below.



- (a) Let $h(x) = f(x)g(x)$. Find $h'(2)$.
 (b) Let $h(x) = f(x)/g(x)$. Find $h'(-4)$.