Math 231 Calculus 1 Fall 20 Midterm 3

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a US letter page of notes, but no other resources.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

l'Hôpital

(1) Find $\lim_{x\to 0} \frac{\cos(7x)-1}{\sin(8x^2)}$. $= \lim_{x\to 0} \frac{-\sin(7x)\cdot 7}{\cos(8x^2)\cdot 16x}$

l'Hôpital

$= \lim_{x\to 0} \frac{-\cos(7x)\cdot 49}{-\sin(8x^2)\cdot 16^2 x^2 + \cos(8x^2)\cdot 16} = \frac{49}{16}$

(2) Consider the function $f(x) = xe^x + 7e^x$.
  (a) Find all critical points of the function.
  (b) Use the second derivative test to attempt to classify them

a) $f'(x) = e^x + xe^x + 7e^x = xe^x + 8e^x$

Solve $f'(x) = 0$: $e^x(x+8) = 0$ $e^x \neq 0$

critical point $\nearrow$ $x = -8$

b) $f''(x) = e^x + xe^x + 8e^x = e^x(x+9)$

$f''(-8) = e^{-8} \cdot 1 > 0 \Rightarrow$ local min

(3) Consider the function $f(x) = \frac{1}{x^2 + x - 2}$. $= \frac{1}{(x-1)(x+2)}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes $x = 1, -2$

$\lim_{x \to \pm\infty} \frac{1}{x^2+x-2} = 0$ horizontal asymptote $y = 0$

b) $f'(x) = -(x^2+x-2)^{-2} \cdot (2x+1)$

solve $f'(x) = 0$: $x = -1/2$ ← critical point

c) $f'(x) > 0$ for $x < -\frac{1}{2}$ so increasing* on $(-\infty, -\frac{1}{2})$.

$f'(x) < 0$ for $x > -\frac{1}{2}$ so decreasing* on $(-\frac{1}{2}, \infty)$.

* where defined

(4) Find the definite integral $\int_0^{\pi} 2x\cos(2x^2)dx$.

sub $u = 2x^2$

$\frac{du}{dx} = 4x$

$$\int_0^{2\pi^2} 2x\cos(u)\,\frac{dx}{du}\,du$$

$$\int_0^{2\pi^2} 2x\cos(u)\,\frac{1}{4x}\,du = \int_0^{2\pi^2} \frac{1}{2}\cos(u)\,du$$

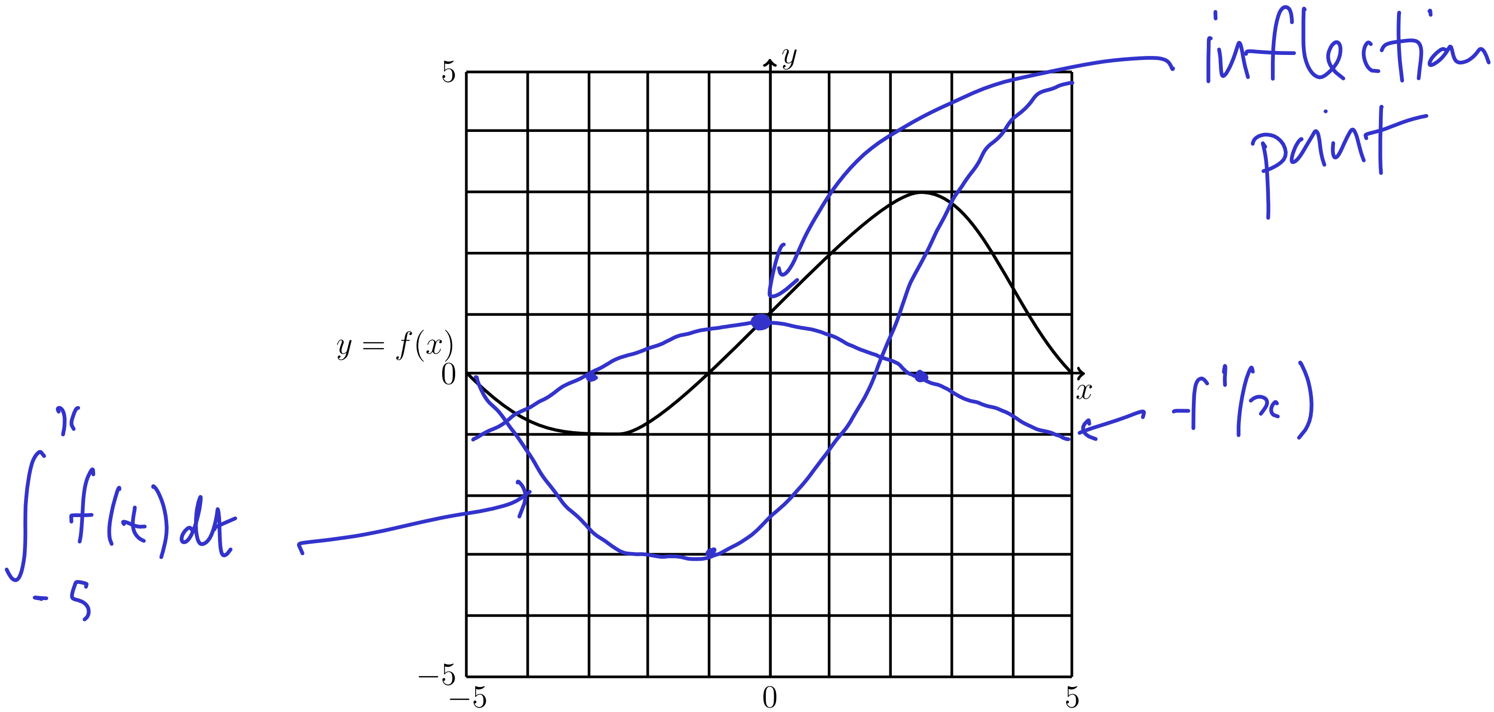
$$= \left[\frac{1}{2}\sin(u)\right]_0^{2\pi^2} = \frac{1}{2}\sin(2\pi^2)$$

(5) Find $\lim_{x \to 0} \dfrac{\tan(2x)}{e^{5x} - 1}$.

l'Hôpital

$$= \lim_{x \to 0} \frac{\sec^2(2x) \cdot 2}{5e^{5x}} = \frac{2}{5}$$

(6) Consider the function $f(x)$ defined by the following graph.



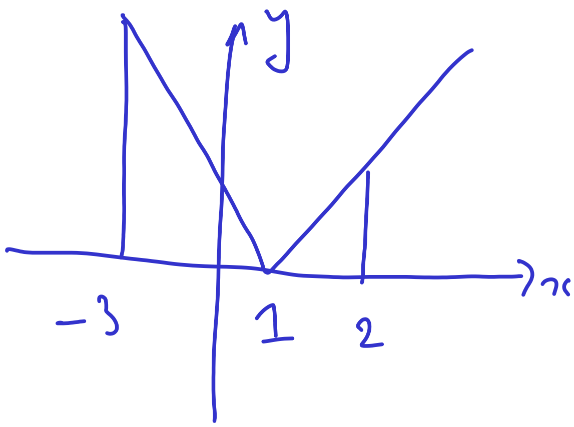
(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection.

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.

(7) Find the area under the graph $y = |x - 1|$ between $x = -3$ and $x = 2$.

Hint: draw a picture.



$$\int_{-3}^{2} |x-1| \, dx$$

$$= \int_{-3}^{1} -x+1 \, dx + \int_{1}^{2} x-1 \, dx$$

$$= \left[-\frac{1}{2}x^2 + x \right]_{-3}^{1} + \left[\frac{1}{2}x^2 - x \right]_{1}^{2}$$

$$= -\frac{1}{2} + 1 - \left(-\frac{9}{2} - 3 \right) + \frac{4}{2} - 2 - \left(\frac{1}{2} - 1 \right)$$

$$\frac{1}{2} + \frac{15}{2} + \frac{1}{2} = 8\frac{1}{2}$$

(8) Find the indefinite integral $\int \frac{8}{1+x^2} - 4\sin(x)dx$.

$$8\int \frac{1}{1+x^2}\,dx - 4\int \sin(x)\,dx$$

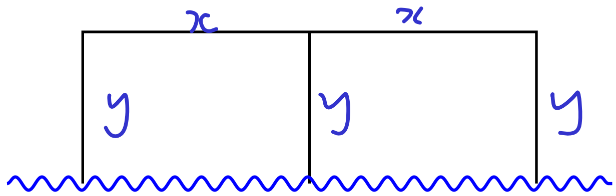
$$8\tan^{-1}(x) + 4\cos(x) + C$$

(9) Find the indefinite integral $\int \frac{6x^3 - 2\sqrt[3]{x}}{\sqrt{x}} dx$.

$$= \int 6x^{5/2} - 2x^{-1/6} \, dx$$

$$= \frac{12}{7} x^{7/2} - \frac{12}{5} x^{5/6} + c$$

(10) You wish to construct two adjacent rectangular fields of equal dimension, sharing a common fence, next to a river, as shown below. If you have 60 feet of fencing, what is the largest area you can enclose?



$2x + 3y = 60$

$A = 2xy$

$$A = (60 - 3y)\,y = 60y - 3y^2$$

$$\frac{dA}{dy} = 60 - 6y$$

$y = 10$

$x = 15$

$A = 300$