Math 231 Calculus 1 Fall 18 Midterm 3a

Name: Solutions

- I will count your best 8 out of the following 10 questions.
- You may use a calculator, and a 3 × 5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) (10 points) Consider the function $f(x) = \frac{1}{x^2 - 5x + 6}$.

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) $\frac{1}{x^2-5x+6} = \frac{1}{(x-3)(x-2)}$ vertical asymptotes $x = 2, 3$.

$\lim_{x \to \infty} \frac{1}{(x-3)(x-2)} = 0 \quad \lim_{x \to -\infty} \frac{1}{(x-3)(x-2)} = 0$

two horizontal asymptotes $y = 0$

b) $f'(x) = -(x^2-5x+6)^{-2} \cdot (2x-5) = \frac{5-2x}{(x^2-5x+6)^2} = 0 \Rightarrow x = \cancel{3/5}\ 5/2$

$+ \quad | \quad - \qquad -(2x-5) \Rightarrow \cancel{f}$

$+ \quad \cancel{3/5}\ 5/2 \quad - \qquad f'(x)$

$(-\infty, 5/2)$ increasing

$(5/2, \infty)$ decreasing

really: ~~$(5/2, 2) \cup (2,3) \cup (3, \infty)$~~.

$(-\infty, 2) \cup (2, 5/2)$ increasing

$(5/2, 3) \cup (3, \infty)$ decreasing

(2) (10 points) Consider the function $f(x) = x^2 \ln(x)$, $x > 0$.
(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them.

a) $f'(x) = 2x\ln(x) + x^2 \cdot \frac{1}{x} = 2x\ln(x) + x$

solve $f'(x) = 0$: $x(2\ln(x)+1) = 0$ $x = 0$ ← ignore $x > 0$

$x = e^{-1/2}$

b) $f''(x) = 2\ln(x) + 2x \cdot \frac{1}{x} + 1 = 2\ln(x) + 3$

$f''(e^{-1/2}) = 2\ln(-\frac{1}{2}) + 3) = -1 + 3 = 2 > 0$ minimum.

(3) (10 points) Find $\lim_{x \to 0} \frac{e^{3x} - 1}{\sin(2x)}$. $\frac{0}{0}$ indeterminate form

l'H: $= \lim_{x \to 0} \frac{3e^{3x}}{2\cos(2x)} = \frac{3}{2}$

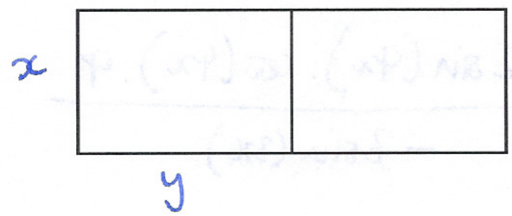
(4) (10 points) Find $\lim\limits_{x \to 0} \dfrac{\sin^2(4x)}{\cos(3x) - 1}$. $\frac{0}{0}$ indeterminate fam.

l'H: $= \lim\limits_{x \to 0} \dfrac{2\sin(4x) . \cos(4x) . 4}{-3\sin(3x)}$ $\frac{0}{0}$ indeterminate fam

l'H: $= \lim\limits_{x \to 0} \dfrac{8 . \left(\cos(4x) . 4\cos(4x) \overset{-}{\cancel{+}} \sin(4x) . \sin(4x) . 4 \right)}{-9\cos(3x)}$

$= -\dfrac{\cancel{8}32}{9}$

(5) You wish to construct two adjacent rectangular fields of equal dimension, sharing a common fence, as shown below. If you have 360 feet of fencing, what is the largest area you can enclose?



$$A = 2xy$$

$$L = 3x + 4y = 360 \qquad y = \frac{360-3x}{4}.$$

$$A = 2x\left(\frac{360-3x}{4}\right) = \frac{360x - 3x^2}{4}.$$

$$\frac{dA}{dx} = \frac{360-6x}{4} = 0 \Rightarrow x = 60$$

$$A = \frac{2.60(360-180)}{4} = 30.60 =$$

$$= 1800 \text{ ft}^2.$$

$2x^{1/3}$

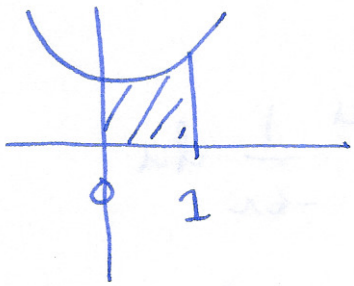
(6) (10 points) Find the indefinite integral $\int 3x^2 - 2\sqrt[3]{x}\, dx$.

$x^3 - 2 \cdot \frac{3}{4} x^{4/3} + C$

(7) (10 points) Evaluate the indefinite integral $\int 2e^x - 3\cos(x)\, dx$.

$2e^x - 3\sin(x) + C$

(8) (10 points) Find the area under the graph $y = 2x^4 + 1$ between $x = 0$ and $x = 1$.



$$\int_0^1 2x^4 + 1\,dx = \left[\frac{2}{5}x^5 + x\right]_0^1$$

$$= \frac{2}{5} + 1 = \frac{7}{5}$$

(9) (10 points) Find the indefinite integral $\int xe^{-3x^2}\,dx$.

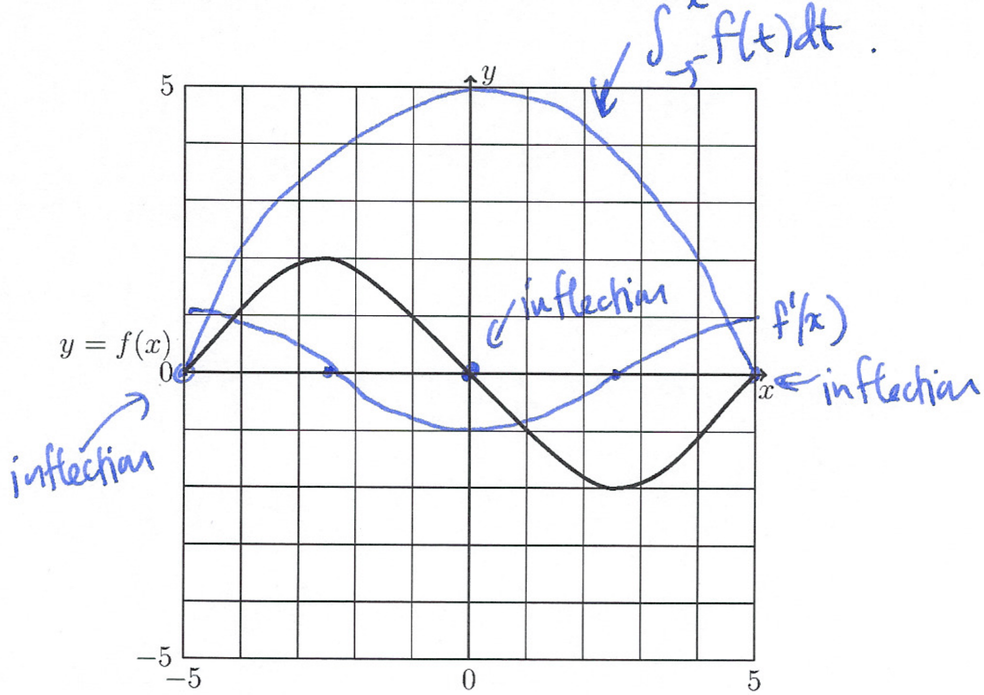
$u = -3x^2$

$\frac{du}{dx} = -6x$

$\int xe^u \cdot \frac{dx}{du}\,du = \int xe^u \cdot \frac{1}{-6x}\,du$

$= -\frac{1}{6}\int e^u\,du = -\frac{1}{6}e^u + c = -\frac{1}{6}e^{-3x^2} + c$

(10) (10 points) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection. $-5, 0, 5.$

(c) Sketch the graph of $\int_{-5}^{x} f(t)\, dt$.