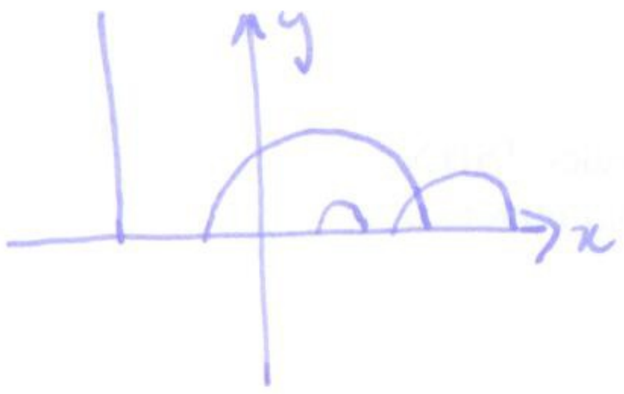
§8 Hyperbolic geometry

A concrete model: upper half space $\{(x,y) \in \mathbb{R}^2 \mid y>0\}$



points ↔ points in upper half space

lines ↔ lines or circles perpendicular to x-axis. (semicircles / center on x-axis)

[angles ↔ Euclidean angles].

- there is a unique line between any two points.

- given a line L, and a point P not on the line, there are infinitely many lines through p which do not hit L.

parallel postulate fails.

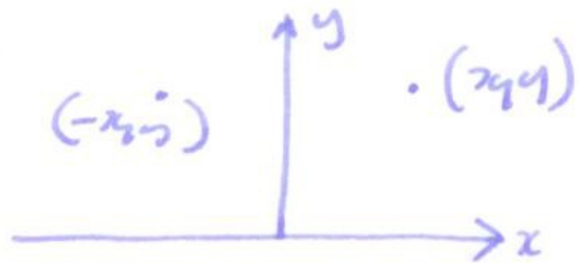
Q: how do we describe the isometries? recall: isometries take straight lines to straight lines. example translation along x-axis.

$(x,y) \mapsto (x+a, y)$

$z \mapsto z+a \quad a \in \mathbb{R}$.

§8.2 Complex conjugation

reflection in y-axis:



$(x,y) \mapsto (-x,y)$

↓ notation $z = x+iy \mapsto -x+iy = -\bar{z}$