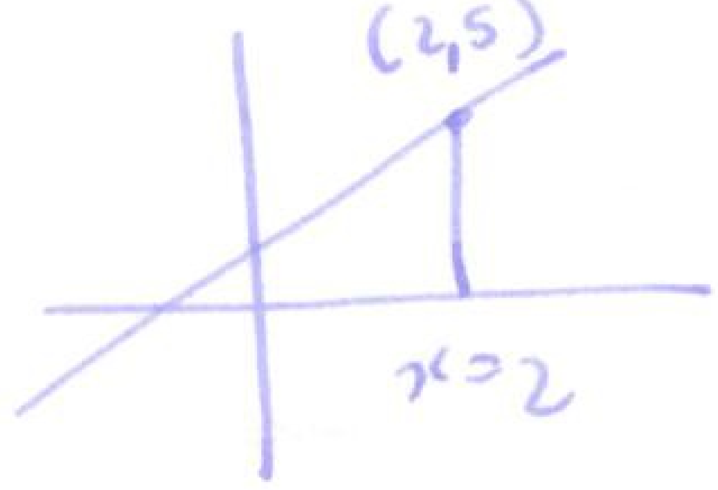
c) $\lim_{x \to 2} 2x+1 = 5$



want to show $|f(x)-5|$ close to 0 for x close to 2

$|2x+1-5| = |2x-4| = 2|x-2|$

x close to 2 $\Leftrightarrow$ $|x-2|$ close to 0, so $|f(x)-5|$ close to zero.

<u>Thm</u> for any constants k, c

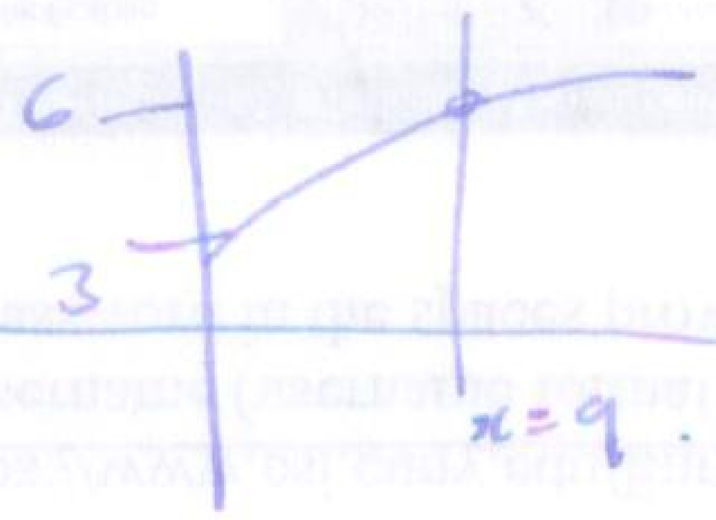
a) $\lim_{x \to c} k = k$ b) $\lim_{x \to c} x = c$

<u>Investigating limits</u>: try · drawing a picture

· estimating/calculating close values.

<u>Example</u> $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$ problem: can't just plug in $x=9$.

<u>try</u>: · drawing a picture:



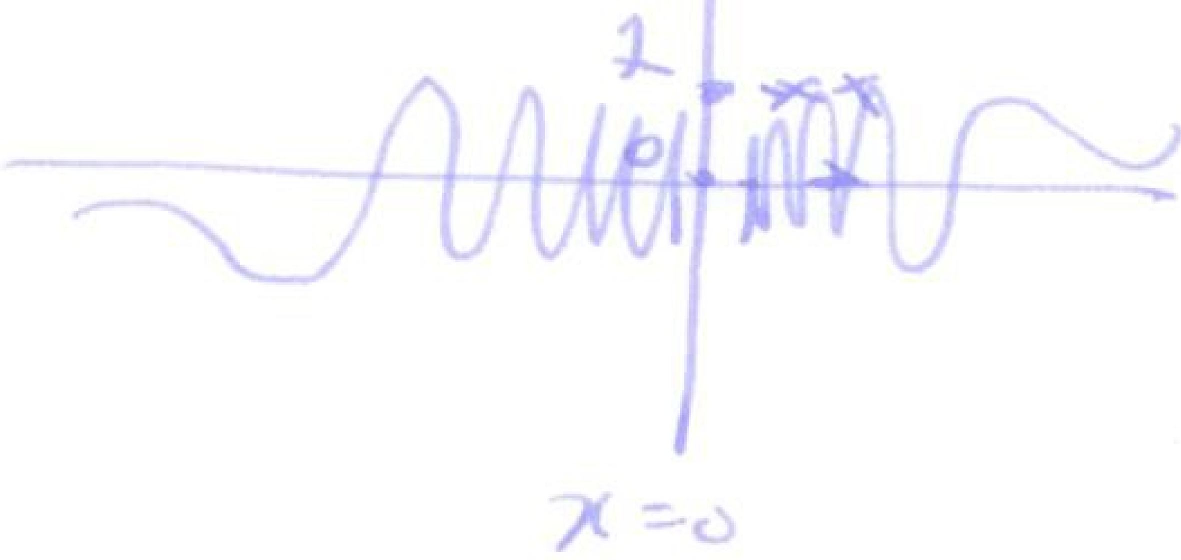
looks like 6.

· compute

x	8.9	9.1	8.99	9.01
$\frac{x-9}{\sqrt{x}-3}$	5.98329	6.016	5.99833	6.00166

· algebraically: we'll do this later.

Bad example : no limit $f(x) = \sin(\frac{1}{x})$

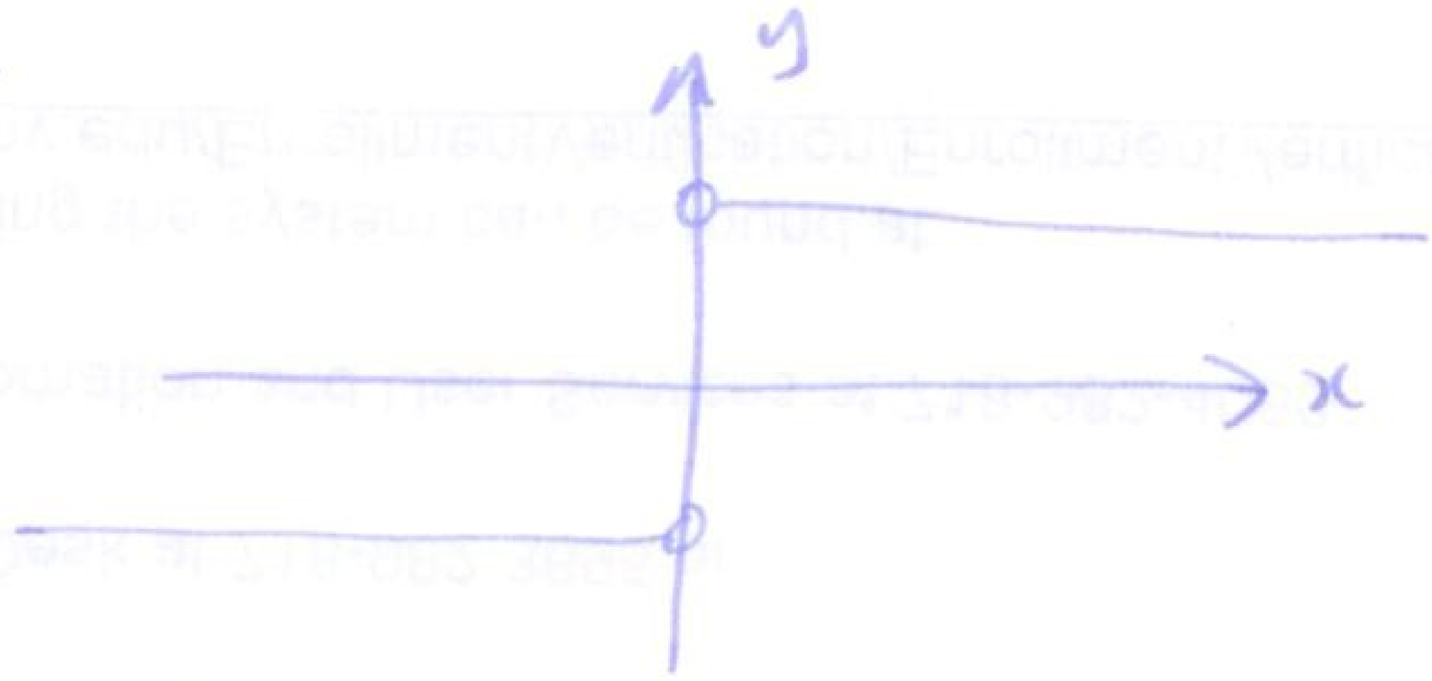


note $f\left(\frac{1}{2\pi n}\right) = \sin(2\pi n) = 0$

$f\left(\frac{1}{2\pi n + \pi/2}\right) = \sin\left(2\pi n + \frac{\pi}{2}\right) = 1$

One sided limits

$f(x) = \frac{x}{|x|}$



$= \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$

sometimes useful to distinguish left from right

$\lim_{x \to 0+} f(x)$ means right limit

$\lim_{x \to 0-} f(x)$ means left limit

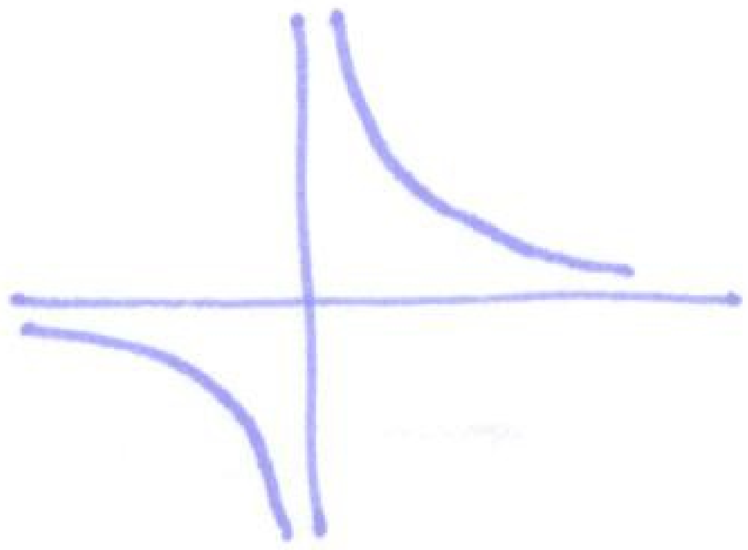
note in order for $\lim_{x \to 0} f(x)$ to exist both $\lim_{x \to 0+} f(x)$ and $\lim_{x \to 0-} f(x)$ must exist and be equal.

Infinite limits

We say $\lim_{x \to c} f(x) = +\infty$ if $f(x)$ becomes arbitrarily large and positive as $x \to c$.

we say $\lim_{x \to c} f(x) = -\infty$ if $f(x)$ becomes arbitrarily negative as $x \to c$

<u>Example</u> $f(x) = \frac{1}{x}$

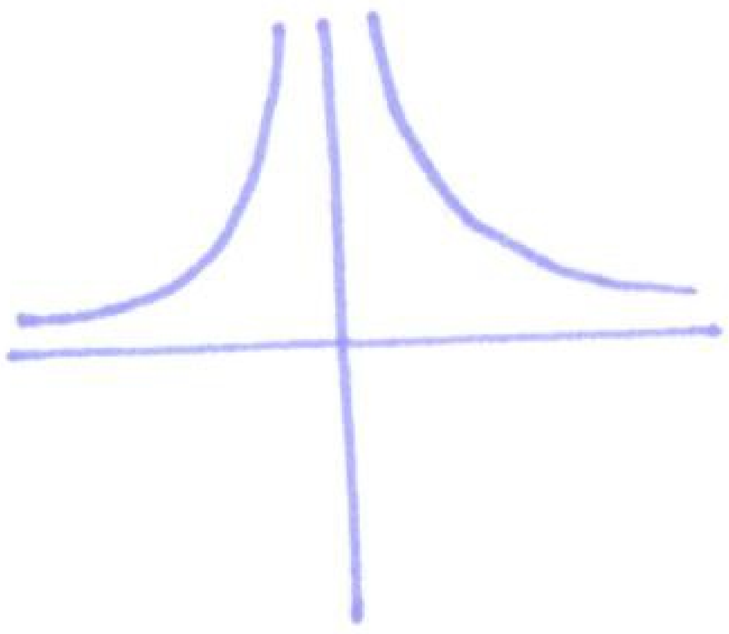


$\lim_{x \to 0+} f(x) = +\infty$

$\lim_{x \to 0-} f(x) = -\infty$

$\left.\right\}$ $\lim_{x \to 0} f(x)$ does not exist!

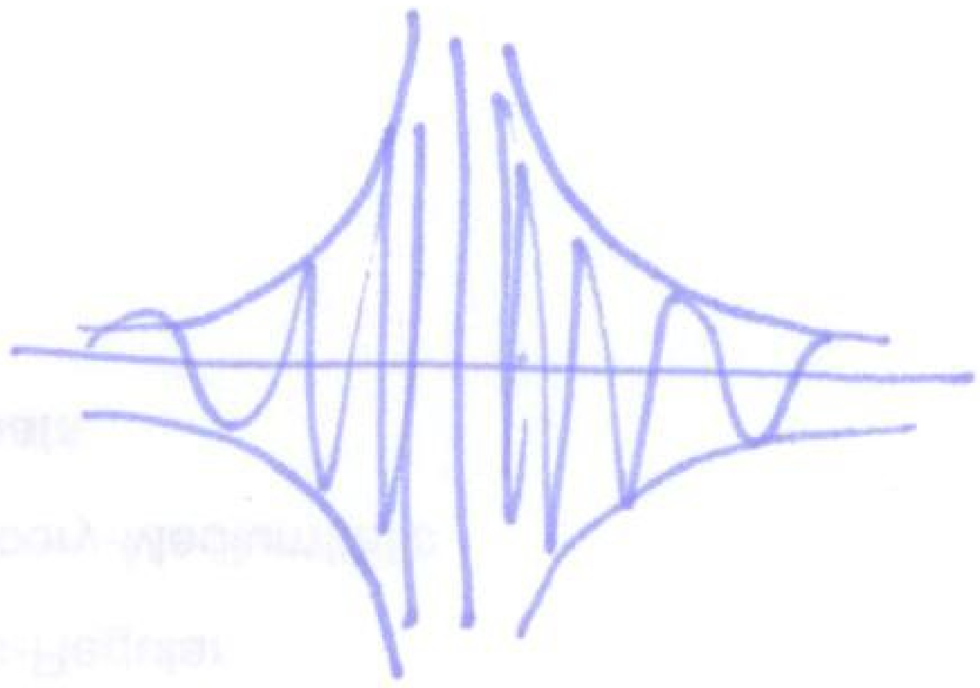
$f(x) = \frac{1}{x^2}$



$\lim_{x \to 0+} f(x) = +\infty = \lim_{x \to 0-} f(x)$

so $\lim_{x \to 0} f(x) = +\infty$

<u>Bad example</u> $f(x) = \frac{1}{x} \sin\left(\frac{1}{x}\right)$



no limit as $x \to \infty$.

§2.3 Basic limit laws

<u>Th^m</u> Assume $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist and are finite.

1) sums: $\lim_{x \to c} \left(f(x) + g(x) \right) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$

2) constant multiple: $\lim_{x \to c} k f(x) = k \lim_{x \to c} f(x)$ $\quad k$ constant

3) products: $\lim_{x \to c} \left(f(x) g(x) \right) = \left(\lim_{x \to c} f(x) \right) \left(\lim_{x \to c} g(x) \right)$

4) quotients: if $\lim_{x \to c} g(x) \neq 0$ then $\lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$